6.1 Introduction

The advancement in construction materials and effective utilization of the available raw materials have lead to use of various supplemental cementitious materials in concrete for the sustainability of the concrete industry. Commonly used supplemental cementitious materials used in cement concrete which are of pozzolanic in nature are fly ash, micro silica, metakaolin, rice husk ash, ground granulated blast furnace slag etc., From the brief review of the available literature (section 2.6), it is observed that, investigations on fly ash based concrete are concentrated on mechanical and Mode – I fracture characteristics of such concrete.

This fly ash based concrete when used in structures which are subjected to high intensity concentrated loads, its punching shear resistance capacity becomes one of the controlling factor for the service of the structure. Hence the fracture characteristics of fly ash based concrete in Mode –II fracture is to be studied.

Hence in this thesis, fly ash is considered as a supplemental cementitious materials by partially replacing the ordinary Portland cement by mass in concrete in the percentages of 0, 5, 10, 15, 20, 25 and 30 and its Mode –II fracture characteristics are analyzed. For this, DCN (Double Centered Notched) specimen geometry [Fig.1.4] developed by Prakash desayi et al.[1] and Bhaskar Desai [2], is considered to study the Mode – II fracture characteristics of concrete
by partially replacing the cement with fly ash in the above proportions. Along with this Mode – II fracture, the strengths of this concrete in direct compression and split tension are also studied.

The literature review on the fly ash concrete is presented in section 2.6. The organization of this chapter is arranged as follows. The details of fly ash and its reaction when used in concrete are presented in section 6.2, the details of experimental work and experimental observations are presented in section 6.3, the results and discussions of experimental investigations are presented in section 6.4, probabilistic analysis on Mode – II fracture concrete with fly ash are presented in section 6.5, results and discussions of the probabilistic analysis are presented in section 6.6, study of the ratio between Mode- I and Mode – II fracture energies (experimental and probabilistic) are presented in section 6.7, and finally summery and conclusions of the experimental and probabilistic analysis are given in section 8.3

6.2.1 Fly ash

Fly ash is also known as pulverized fuel ash (PFA), which is a residue generated from combustion of pulverized coal as a fuel in electric power generating plants. During the combustion of the pulverized coal, the mineral impurities present in the coal (clay, feldspar, quartz, and shale) fuse in suspension and float out of the combustion chamber along with hot exhaust gases [Siddique and Khan][90]. As this fused material rises, it gets naturally cooled and solidifies into a fine grained, powdery, spherical glassy particles called fly ash. This material will be collected from the exhaust gases before it comes out from the chimneys by particle filtration equipments such as electrostatic precipitators or bag filters. Fly ash accounts for 75 to 85 percent of the total coal ash, and the remainder is collected as bottom ash or boiler slag. The combustion of pulverized coal not only produces the CO₂ into the atmosphere, but also causes the problem of safe disposal of the fly ash, which has combined effect on the global environment.
The utilization of fly ash in construction industry has started from few decades back, with wide range of applications ranging from manufacture of concrete (as a partial substitute for cement or sand), controlled low strength materials, stabilization of soft soils, liquid waste stabilization, roller – compacted no fines concrete, mineral filler in asphaltic concrete, high volume fly ash concrete, manufacturing of light weight aggregate (fly ash pellets to be used as light weight coarse aggregate in concrete), grout and flowble fill production, fly ash concrete bricks, etc., Being a pozzolanic material with silica content, it reacts readily with calcium hydroxide of hydrated cement and forms a compound similar to that produced by the hydrated cement, but with some variations in composition. Due to this, it has been used as an important ingredient in concrete in varying percentages by weight of cementitious material component, thereby reducing the Portland cement utilization in concrete, this also reduces the emission of carbon dioxide during the calcination process in the manufacture of Portland cement clinker, which is a major contributor to greenhouse gas emissions that are implicated in global warming and climate change. (CaCO₃ on heating gives CaO+CO₂)

The utilization fly ash in concrete as an alternate cementitious material component, has many advantages in both of its fresh and hardened states. It mainly reduces the cost of concrete and enhances the durability of structures along with effective utilization of the available resources. The utilization of fly ash in concrete varies widely depending on the application of concrete, properties of the fly ash, specification limits and the geographic location and climatic conditions. Generally, to reduce the heat of hydration in mass concrete structures such as massive foundations, dams and bridge piers etc., higher levels of low calcium fly ash (class F) will be used (more than 50% replacement of cement with fly ash in concrete). But, at higher replacement levels problems may be encountered due to increase in concrete setting time and slower rate of
strength development in concrete due to slower pozzolanic reaction of fly ash, ultimately causing delay in the rate of construction. These drawbacks are particularly pronounced in cold weather concreting. Also the increased risk of carbonation induced corrosion in fly ash concrete is generally a concern when high levels of fly ash are used in low grade, poorly cured concretes, and concretes with low cover depths over the reinforcement steel.

The properties of fly ash produced from thermal power plants will vary due to the following influencing factors

1. Type and mineralogical composition of the coal.
2. The relative amounts of incombustible material in the coal.
3. Degree of coal pulverization.
4. Type of furnace and oxidation conditions.
5. The manner in which fly ash is collected, handled and stored before its use.

The size of the fly ash particles primarily depends upon the type of dust collecting equipment. The shape of fly ash particles are generally spherical and ranges from 1µ to 500µ. ASTM C618 [91] classifies the fly ash as class C and class F

Class C fly ash

These are generally produced from burning of younger formations of coal such as lignite or sub-bituminous coal. The CaO content varies from 11.6% to 29.0% [90]. These fly ashes contain predominantly calcium alumino-silica glass which is highly reactive. Crystalline phases in Class C fly ash includes quartz, lime, mullite, gehlenite, anhydrite and cement materials such as C₃A, C₂S and C₄A₃S. Unlike class F, class C fly ash does not require an activator (such as Ca OH which is required for class F fly ash).
Class F fly ash

These are normally produced from burning older formations of coal such as anthracite or bituminous coal. This class of fly ash exhibits pozzolanic property but rarely, if any, self-hardening property due low lime (CaO) content (0.7 to 7.5%)[90]. They contain predominantly (>70%) noncrystalline silica which is the determining factor for pozzolanic activity. The crystalline minerals in Class F fly ash generally composed of quartz, hematite, mullite and magnetite. Possessing pozzolanic properties, the glassy silica and alumina of Class F fly ash requires a cementing agent, such as Portland cement, quicklime or hydrated lime, with the presence of water in order to react and produce cementitious compounds.

The requirements specified by ASTM C618 [91] for class C and class F fly ashes to use as a mineral admixture in ordinary Portland cement concrete are presented in Table 6.1

The properties of fresh and durability of hardened concrete are strongly influenced by the incorporation of the fly ash into the mixture. The extent to which fly ash affects these properties is dependent not only on the level and the composition of the fly ash, but also on other parameters including the composition and proportion of the other ingredients of the concrete, the exposure conditions during and after placement etc.,
### Table 6.1 The required specifications specified by ASTM for class C and class F fly ash

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Class C</th>
<th>Class F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical requirement, $\text{SiO}_2 + \text{Al}_2\text{O}_3 + \text{Fe}_2\text{O}_3$, minimum %</td>
<td>50.0</td>
<td>70.0</td>
</tr>
<tr>
<td>$\text{SO}_3$, maximum %</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Moisture content, maximum %</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Loss on ignition, maximum %</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Amount retained when wet sieved on 45µm sieve, maximum %</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>Pozzolanic activity index, with Portland cement at 28 days, minimum % of control</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Pozzolanic activity index with lime at 7 days, minimum MPa</td>
<td>-</td>
<td>5.5</td>
</tr>
<tr>
<td>Water requirement, maximum % of control</td>
<td>105</td>
<td>105</td>
</tr>
<tr>
<td>Autoclave expansion or contraction, maximum %</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Specific gravity, maximum variation from average value in percentage</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Percentage retained on 45µm sieve, maximum variation, percentage points from average</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

#### 6.2.2 Portland cement hydration [Neville][92]

The setting and hardening process of concrete, which occurs after the four components i.e., coarse aggregate, fine aggregate, cement and water are mixed together, is largely due to the
reaction between the components of cement and water. The other two components, coarse aggregate and fine aggregate, are more or less inert as far as setting and hardening is concerned. The reactions taking place in a freshly mixed concrete and the process by which it gains strength is as follows.

(a) **Hydration of calcium silicate**

When hydration takes place, C₃S and C₂S undergoes hydrolysis producing a calcium silicate of lower basicity, ultimately C₃S₂H₃ with released lime separating out as Ca(OH)₂.

\[
2\text{C}_3\text{S} \quad + \quad 6\text{H} \quad \rightarrow \quad \text{C}_3\text{S}_2\text{H}_3 \quad + \quad 3\text{Ca(OH)}_2
\]

Tricalcium silicate + water \quad C-S-H + calcium hydroxide

\[
2\text{C}_2\text{S} \quad + \quad 4\text{H} \quad \rightarrow \quad \text{C}_3\text{S}_2\text{H}_3 \quad + \quad \text{Ca(OH)}_2
\]

dicalcium silicate + water \quad C-S-H + calcium hydroxide

(b) **Hydration of tricalcium aluminate and the action of gypsum:**

The reaction of pure C₃A with water is leads to immediate stiffening of the paste, to prevent this, gypsum (CaSO₄.2H₂O) is added to cement.

The gypsum (CaSO₄.2H₂O) which is added to cement klinker, for preventing flash setting, reacts with C₃A to form insoluble calcium sulfoaluminate (3CaO.Al₂O₃.3CaSO₄.31H₂O) known as ettringite. The number of molecules may be 31 or 32 depending on the ambient vapour pressure.

\[
\text{C}_3\text{A} \quad + \quad 3\text{CS} \quad + \quad 26\text{H} \quad \rightarrow \quad \text{C}_3\text{A} \quad (\text{CS})_3 \quad \text{H}_{32}
\]

Tricalcium aluminate \quad gypsum \quad water \quad ettringite

(c) **Hydration of tetra calcium alumino ferrite and the action of gypsum**

Gypsum reacts with C₄AF to form calcium sulfoferrite as well as calcium sulfoaluminate (ettringite) i.e., C₄AF forms hydration products similar to that of C₃A, where iron substitutes partially for alumina in the crystal structure of ettringite and monosulpho aluminate hydrate.
(d) Alkali – silica reaction

In case of ordinary Portland cement the alkalis present in the form of Na$_2$ and K$_2$, alkali hydroxides formed during the hydration process of cement reacts with active silica constituents in the aggregates, forming alkali-silicate gel either in planes of weakness or pores in the aggregates (where reactive silica is present) or on the surface of the aggregate particles, in the latter case a characteristic altered surface zone is formed. This may destroy the bond between the aggregate and the surrounding hydrated cement paste.

6.2.3 Pozzolanic activity and hydration

Fly ash is pozzolanic material. According to ASTM C618 – 94a [91], pozzolana is treated as “a siliceous or siliceous and aluminous material which in itself possesses little or no cementitious value but which, in finely divided or powdered form, and in the presence of moisture, chemically reacts with calcium hydroxide at ordinary temperatures to form compounds that possess cementitious properties”. It is essential that pozzolana be in a finely divided state as it is only then that silica can combine with calcium hydroxide (produced by the hydrating Portland cement).

The major reaction that takes place is between the reactive silica of the pozzolana and calcium hydroxide producing calcium silicate hydrate (C-S-H). This main reaction for fly ash is known as pozzolanic reaction. The alumina in the pozzolana may also react with calcium hydroxide and other compounds in the mixture to form similar products.

\[
\text{CaOH} + S \text{ (silica from fly ash constituent)} \quad \rightarrow \quad \text{C-S-H}
\]

At high pH values, the silicate ion in the fly ash becomes soluble to react with excess calcium in the solution form, which liberated during the hydration of cement. According to Ferry et al.
[51] in case of class F fly ash, the glassy mineral in fly ash is broken down only when the pH value of the pore water is at least about 13.2, and the increase in the alkalinity of the pore water requires that a certain amount of hydration of the Portland cement in the mix has taken place. Moreover, the reaction products of Portland cement precipitate on the surface of the fly ash particles, which act as nuclei. When the pH of the pore water becomes high enough, the products of reaction of the fly ash are formed on the fly ash particles and in their vicinity.

For a given particular construction, there will be an optimum level of utilization of fly ash replacement as a cementitious material (whether ASTM C618 [91] Class C-lignitic type or Class F-bituminous type) which will maximize the technical and economical benefits to the structure and environmental benefit by utilizing the fly ash without significantly effecting the rate of construction or impairing the long term performance of the structures. The amount of fly ash to be used as a supplementing cementing material in concrete varies widely depending on the properties of the fly ash, type of the structure and its exposure environment. Mixture proportioning and trial batches are critical to obtain concrete with the desired fresh and hardened properties. Fly ash may be introduced in concrete as a blended cement containing fly ash or introduced as a separate component at the mixing stage.

Generally mixing of fly ash is done in concrete as a separate component at the concrete batching and mixing plants. This allows the flexibility of getting required mixture proportions to obtain the required concrete properties for the particular application and performance.
6.3 Experimental investigations

6.3.1 Materials used

The materials used in the experimental programme are presented in Fig.6.1

Fig.6.1  Super plasticizer, cement, fly ash, fine aggregate and coarse aggregate

Cement

Ordinary Portland cement of 53 grade confirming to I.S:12269-1987 [79] of ultratech brand has been used. Its chemical compositions and chemical compounds are presented in Table.6.2 and Table 6.3 (Obtained from the Ultratech cement company, Tadipatri, Anantapur, A.P)
Table 6.2 Chemical composition of ultratech 53 grade ordinary Portland cement

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Contents in the cement</th>
<th>% by mass</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Silicon dioxide, SiO₂</td>
<td>17-25 %</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Aluminum trioxide, Al₂O₃</td>
<td>4-8%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Ferric trioxide, Fe₂O₃</td>
<td>0.5-0.6 %</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Calcium oxide, CaO</td>
<td>61-63 %</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Magnesium oxide, MgO</td>
<td>0.1-4.0 %</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Sulphur trioxide, SO₃</td>
<td>1.3-3.0 %</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Sodium oxide + Potassium oxide, Na₂O+K₂O</td>
<td>0.4-1.3 %</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Chlorine, Cl</td>
<td>0.01-0.1%</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>IR</td>
<td>0.6-1.75 %</td>
<td></td>
</tr>
</tbody>
</table>

In addition to the above, other minor compounds such as TiO₂, Mn₂O₃, and N₂O in small quantities are present. The extent of chemical compounds in the ultratech 53 grade ordinary Portland cement are presented in the following table.

Table 6.3 Extent of chemical compounds present in the ultratech 53 grade ordinary Portland cement

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Compound</th>
<th>% by mass</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tri calcium silicate, C₃S</td>
<td>48-52 %</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Di calcium silicate C₂S</td>
<td>22-26 %</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Tri calcium Aluminate, C₃A</td>
<td>6-10 %</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Tetra calcium Alumino FerriteC₄AF</td>
<td>13-16 %</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Freelime</td>
<td>1-2 %</td>
<td></td>
</tr>
</tbody>
</table>
The test results concluded on cement are as follows.

Initial setting time = 35 minutes

Final setting time = 500 minutes

Specific gravity = 3.2

Fineness = 3 percent

Normal consistency = 33.5 percent

In order to avoid the possible variation in the properties of cement from various batches all the specimens are prepared from the same batch of cement.

**Fly ash**

Fly ash used in this experimental work is obtained from Rayalaseema Thermal Power Plant (RTPP) at Muddanuru, Cuddapah District, (A.P). The chemical composition and physical properties of the fly ash obtained from (RTPP) are as given in Table. 6.4 which comes under the category of Class F fly ash. The particle size distribution curve of this fly ash is shown in Fig.6.2

![Fig.6.2 Particle size distribution curve of fly ash](image)
Table. 6.4 Chemical composition of fly ash used in the experimental investigation (obtained from Rayalaseema Thermal Power Plant, Muddanuru, Cuddapah, A.P)

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Contents in the fly ash sample</th>
<th>% by mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SiO$_2$+Al$_2$O$_3$+Fe$_2$O$_3$</td>
<td>89.86</td>
</tr>
<tr>
<td>2</td>
<td>Silicon dioxide , SiO$_2$</td>
<td>56.93</td>
</tr>
<tr>
<td>3</td>
<td>Magnesium oxide , MgO</td>
<td>1.20</td>
</tr>
<tr>
<td>4</td>
<td>Total sulphur as Sulphur trioxide, SO$_3$</td>
<td>1.24</td>
</tr>
<tr>
<td>5</td>
<td>Available alkalis as Sodium oxide , Na$_2$O</td>
<td>0.38</td>
</tr>
<tr>
<td>6</td>
<td>Loss on ignition,</td>
<td>0.29</td>
</tr>
<tr>
<td>7</td>
<td>Lead, Pb mg/kg</td>
<td>0.15</td>
</tr>
<tr>
<td>8</td>
<td>Copper , Cu mg/kg</td>
<td>0.18</td>
</tr>
<tr>
<td>9</td>
<td>Zinc , Zn mg/kg</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>10</td>
<td>Manganese , Mn mg/kg</td>
<td>0.42</td>
</tr>
<tr>
<td>11</td>
<td>Boron , B mg/kg</td>
<td>&lt;0.10</td>
</tr>
</tbody>
</table>

Physical properties of fly ash used in the experimental investigation

|   | Fineness – specific surface area, m$^2$/kg                        | 360       |
|   | Specific gravity                                                  | 2.17      |

**Fine aggregate**

Locally available Chitravathi river sand has been used as a fine aggregate, which is free from clay, silt and organic impurities and passing through 4.75 mm size I.S sieve. Its specific gravity
is found to be 2.625. From the sieve analysis results sand, it is observed that sand confirms to zone – I. The particle size distribution curve of sand is shown in Fig.5.2

**Coarse aggregate**

The coarse aggregate consists of locally available machine crushed granite metal passing through 20 mm size IS sieve

**Water**

The water used in this experimental investigation is locally available potable water.

**Super plasticizer**

The super plasticizer used in this experimental investigation is Conplast – SP 430

6.3.2 **Cleaning of Moulds**

Cleaning of the moulds is done as explained section 4.2.2

6.3.3 **Mixing of the ingredients**

The M20 concrete mix has been designed using ISI method (IS:10262-1982)[80] for zero percent replacement of cement. The mix proportion obtained is 1:1.58:2.86 with water cement ratio of 0.5. The design procedure is presented in Appendix-A. Keeping the mass of the binder constant, the cement has been replaced by fly ash in percentages of 0, 5, 10, 15, 20, 25, and 30 by mass. For each percentage replacement of cement considered, the materials are mixed in the standard way. That is, at first the fine aggregate and cementitious materials (i.e., cement and fly ash) are weighed according to their proportion in the concrete mix. Then these materials are mixed thoroughly in dry condition, then this mixture is spread uniformly over the weighed quantity of coarse aggregate and thoroughly mixed in dry condition. Then the measured quantity of water with water cement ratio of 0.5 is added to this dry mix and then mixed thoroughly. For each percentage replacement of cement considered, to have a consistent workability of the mix, a
slump of 100±10 mm is maintained. For this, trial mixes are made in arriving the required slump with the help of superplasticizer, conplast SP – 430 at constant water cement ratio of 0.5. This dosage of super plasticizer is then added in casting the specimens of a given batch.

6.3.4 Casting the specimens

For each percentage replacement, 12 cubes of DCN specimens of size 150x150x150mm with notch depth ratios, (a/w) of 0.3,0.4,0.5 and 0.6 (three samples for each (a/w) ratio); three plain cubes of size 150x150x150 mm and three cylinders of size 150mm diameter and 300 mm height are cast. For each combination of parameters considered in this investigation three nominally similar DCN specimens have been cast and tested. Thus, a total number of 108 specimens are casted in this experimentation (zero percent replacement being common for other experiments). The Fig.1.4 shows the geometrical details of the DCN specimen. Notches of 2mm thick are introduced at one-third points centrally as shown. The notch depths provided are 45, 60, 75 and 90 mm running throughout the width of the specimen. The corresponding shear resisting ligament cross section areas are 31500 mm$^2$, 27000mm$^2$, 22500mm$^2$ and 18000mm$^2$ respectively. It is noted that the (a/w) ratios considered satisfy the condition specified by Reinhardt and Xu [30] namely, the difference in ligament lengths should be large enough so that the effect of inhomogeneity of concrete can be ignored. The distance between notches is kept constant at 50 mm.

For all test specimens, moulds are kept on the vibrating table and the concrete is poured into the moulds in three layers each layer being compacted thoroughly and uniformly with a tamping rod to avoid honey combing. Finally all specimens are vibrated on the table vibrator after filling up the moulds up to the brim. The vibration is effected for seven seconds and it is maintained constant for all specimens and all the other castings. The steel plates forming notches are
removed after three hours of casting carefully and the specimens are neatly finished. However the specimens are demoulded after twenty four hours of casting and are kept immersed in a clean water tank for curing (Fig.4.3). After twenty eight days of curing the specimens are taken out of water and are allowed to dry under shade for few hours.

6.3.5 Loading arrangement and testing of the specimens

All the specimens are white washed and tested in a 2000 kN digital compression testing machine with a uniform rate of loading of 0.1 kN/sec. The loading arrangement for the specimens considered in this experimentation i.e., for cubes, cylinders and DCN specimens are similar to that as considered in section 4.2.5 (Figs.4.4, 4.5, and 4.6).

The companion specimens of cubes are tested for their compressive strength, cylinders are tested for the split tensile strength and DCN specimens are tested for characterizing their in-plane shear fracture behavior. It is to be noted that, though the loading is applied continuously during testing of the DCN specimens, the in-plane displacements of central one third portion if any, are noted at regular loading increments of 10 kN and the initiation and propagation of cracks is observed continuously. The overall test setup for testing of DCN specimens is shown in (Fig.4.6). For measuring the in-plane shear displacement of the central one third portion of the DCN specimen four LVDT’s (least count of 0.01 mm) are arranged as shown in Fig.4.6 The LVDT’s have been arranged such that they represent the localized in-plane shear displacement, there by avoiding noise (Reinhardt and Xu) [30]. To prevent horizontal cracking vertical steel clamps are fixed to the outer ends of the DCN specimen. The LVDT readings are taken at load increments of 10 kN. Then, the average of four LVDT readings is taken to represent the in-plane shear displacement at a given stage of loading. Also, the loads corresponding to the first visual crack initiation load at the end of the of the notch tips ($P_{cr}$) and the ultimate load ($P_u$) are noted.
However, for plotting the load-displacement curve, for specified (a/w) ratio and percentage replacement, the displacement, at a given load level is again averaged over three specimens, so that the most best average load-displacement curves are obtained.

During the experimentation, while in most of the cases the crack initiation is visually observed at the top edge of one or both the notches and then the cracks formed at the bottom edge of the one or both the notches, in some cases visual crack initiation (first) occurred at the bottom edge of the notch(es) and then at the top of the notch(es). This observation could be due to inherent inhomogeneity of concrete. In some of the specimens the failure has occurred due to propagation of crack in the ligaments ahead of the same notch. Thus, formation of a single almost co-planar (though slightly jaggered) crack has led to failure in these specimens. A possible explanation for this observation is hypothesized in chapter 4 based on weakest link hypothesis (section 4.3-4).

Final crack patterns are shown in Fig. 6.3. It can be observed from Fig. 6.3 that there exists a deviation from verticality of the crack path (i.e., the crack propagates in an inclined direction from the tip of the notch). The experimental observations of van Mier

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![Crack patterns with 0% replacement](image1)

![Crack patterns with 5% replacement](image2)
Fig. 6.3 Final crack patterns of cube and DCN specimens at different percentage replacement of cement with fly ash [81, 82] and van Geel [83] on concrete prisms, for both uniaxial and confined compression have shown that the frictional resistance from the loading system will have effect on the direction of the shear crack orientation. Also, Bazant and Pfeiffer [13] showed that crack band propagates sideways when shear force zone is wide and vertically when the shear force zone is narrow (it is noted that the shear force applied in the present experimental programme is over wider area and
hence inclination of cracks). From the experimental final crack patterns it is found that, the cracks ahead of notch tips are more inclined in specimens with lesser (a/w) ratios i.e., a/w=0.3 and 0.4 than those with higher (a/w) ratios i.e., 0.5 and 0.6 considered.

For a given percentage of replacement with fly ash in place of ordinary Portland cement, average values of compressive strength of cube specimens and split tensile strength of cylinder specimens are calculated. The graphical variation of the results of compressive strength and split tensile strengths of concrete are presented in Figs. 6.4 and 6.5.

Fig. 6.4 Variation in cube compressive strength with variation in percentage replacement of cement with fly ash

\[ Y = 26.8276 - 0.16097 X + 0.1027 X^2 - 0.0063 X^3 + 1.02364 \times 10^{-4} X^4 \]

\[ R^2 = 0.9680 \]
Fig. 6.5 Variation in split tensile strength with variation in percentage replacement of cement with fly ash

From the Fig. 6.4 and 6.5, it is observed that the cube compressive strength and split tensile strength increase with increase in percentage replacement with fly ash up to 15 percent (which is also within the range of percentage value as specified by ACI committee 211 [53] for high strength concrete mixes in which ordinary portland cement is replaced with class F fly ash) and then gradually decreases.

For DCN specimens, for each (a/w) ratio and percentage replacement considered, average crack initiation load \( (P_{cr}) \), that is first crack load, and ultimate load \( (P_u) \) are noted. The variation of first visual cracking load \( P_{cr} \) for different combinations of \( (a/w) \), with variation in the percentage replacement of cement by fly ash is shown in Fig.6.6. From this figure it is observed that the crack initiation load increases with the increase in replacement with fly ash content from zero percent to fifteen percent and then gradually decreases. The shear strength corresponding to crack initiation load \( P_{cr} \) for different combinations of \( (a/w) \) and percentage replacements are shown in Fig. 6.7. From this figure it is observed that the shear strength at \( P_{cr} \) attains maximum
value at fifteen percent replacement with fly ash for all (a/w) ratios considered and then gradually decreases. The variation of ultimate load, $P_u$ for different combinations of (a/w) and percentage replacements are shown in Fig. 6.8. From this figure, it is observed that the ultimate load, $P_u$ attains maximum value at 15 percent replacement with fly ash for all (a/w) ratios considered and then gradually decreases with the increase in percentage replacements beyond 15 percent.

Fig. 6.6 Variation in first visual crack load with variation in percentage replacement of cement with fly ash
Fig. 6.7 Variation in shear stress at first crack with variation in (a/w) for different percentage replacements of cement with fly ash.

Fig. 6.8 Variation in ultimate load with variation in percentage replacement of cement with fly ash.

6.3.6 Load verses in-plane shear displacement behavior

An attempt has been made in this investigation to study the variations in Mode - II fracture energy of concrete with varying percentage replacements of cement with fly ash. In order to carry out this study, experimentally determined load-displacement curves of DCN specimens are
used. As already indicated, for a given combination of (a/w) ratio and percentage replacement, there are three nominally similar specimens. At every 10 kN load increment, the mean value of displacement of the three specimens is computed. Also the average visual crack initiation load \( (P_{cr}) \) and ultimate load \( (P_u) \) (of three nominally similar specimens) are considered for drawing the load displacement curve. Thus, for a given combination of parameters considered (for a given notch depth ratio and percentage replacement with fly ash) one load-displacement curve is obtained. The load-displacement curve is shown in Fig. 6.9, typically for \((a/w) = 0.3\) and percentage replacement of 5.

![Graph showing load-displacement curve](image)

**Fig. 6.9** Variation between load and displacement at 5 percent replacement of cement with fly ash and \((a/w)=0.3\)

Also a set of load-displacement curves obtained in this investigation are presented for each \((a/w)\) ratio considered in Fig.6.10 a-d. From the load-displacement curves (viz., Fig. 6.10 a-d), it is observed that due to high initial rigidity, the specimens are taking the initial load without undergoing initial deformation. However, in the computation of work of fracture, the load -
displacement curve is considered to pass through the origin at zero load. Also, as reported in Reinhardt and Xu [30] and by Sih[84], the micro-cracking

![Graph 1](image1.png)

6.10.a- Load Vs displacement variations at different % replacement of cement with fly ash and a/w=0.3

![Graph 2](image2.png)

6.10.b- Load Vs displacement variations at different % replacement of cement with fly ash and a/w=0.4

![Graph 3](image3.png)

6.10.c- Load Vs displacement variations at different % replacement of cement with fly ash and a/w=0.5

![Graph 4](image4.png)

6.10.d- Load Vs displacement variations at different % replacement of cement with fly ash and a/w=0.6

Fig.6.10.a-d. Load – displacement curves for 0%, 5%, 10%, 15%, 20%, 25% and 30% replacements of cement with fly ash and (a/w) ratios of 0.3, 0.4, 0.5 and 0.6
starts at a load approximately equal to fifty percent of first visual cracking load. This is evident from the onset of nonlinearity in the load-displacement curves (Fig.6.10) around this load.

6.3.7 Determination of experimental fracture energy

As defined earlier in section 4.2.7, the area under the load-displacement curve divided by the shear resisting cross section area of the ligaments above and below the notches gives the experimental fracture energy in Mode – II (G_{II\!F}) fracture of DCN specimens. Thus, in the present investigation, the Mode - II fracture energy is obtained by evaluating the area under the load-displacement curve (hatched area shown in Fig.6.9) divided by the corresponding shear resisting area (cross section area of ligaments). A best fit polynomial equation is fitted to the experimentally obtained load-displacement curve. The area under this curve is computed using ORIGIN software.

From the plot of the experimental fracture energy (G_{II\!F}) versus (a/w) ratio (Fig.6.11), for each percentage replacement of the fly ash content, it is observed that there is no significant variation in the fracture energy (G_{II\!F}) with variation in (a/w) ratio for a given percentage the percentage replacement with fly ash. A similar observation was made by Reinhardt and Xu [30] on edge notched specimens with different ligament lengths (based on this observation, they reported average fracture energy G_{II\!F} as a representative quantity). Therefore, the average fracture energy of DCN specimens, for a given percentage replacement in the cement content, corresponds to a specimen with average shear resisting area (i.e., 24750 mm^2). From the Fig.6.12, it is also noted that for all (a/w) ratios considered, the fracture energy increases with the increase in percentage replacement up to 15 percent and then decreases gradually.
Fig. 6.11 Variation of experimental fracture energy (G_{IIF}) with respect to (a/w) ratio at different percentage replacement with fly ash

Fig.6.12 Variation in experimental fracture energy G_{IIF} with variation in percentage replacement with fly ash

6.3.8 Computed and measured shear strains at P_{cr}/2

For average stress – strain behavior of concrete (made with partially replacing the
cement with fly ash), ahead of crack tip of DCN specimens, two dimensional pure shear condition is assumed to estimate the shearing strain. The load – displacements of DCN specimens are measured using the LVDTs over a gauge length of 100 mm.

A graph is plotted from the data of computed and measured strains at $P_{cr}/2$ (Fig. 6.13), for all percentage replacements and $(a/w)$ ratios considered. The sample calculations involved are presented in Appendix-D.1, typically for the case of a DCN specimen with $(a/w) = 0.4$ and percentage replacement equal to 10. A line of equality is also shown in Fig. 6.13. From this figure it is observed that, except for $(a/w)$ ratios 0.4 and 0.5 for zero percent replacement (which may due to initial problems encountered in the experimental observations), the measured strains are slightly higher than the computed strains.

![Graph comparing computed and measured shear strains](image)

Fig. 6.13 Comparison between computed and measured shear strains at an applied load level of $P_{cr}/2$
6.4 Results of experimental observations and discussion

From the plot of variation of cube compressive strength ($f_{cu}$) with percentage of fly ash replacing the cement (Fig. 6.4), it is observed that the cube compressive strength attains the maximum value for 15% replacement and afterwards it gets decreased. This is also as in reports of Thomas [59], ACI committee report 211[53] recommendation limits of 15 to 25 percent. The value of $f_{cu}$ even for 30 percent replacement is little higher than zero percent replacement, suggesting that fly ash can be used as a replacement for cement up to 30 percent and thus meeting the sustainability goals (Glavind and Petersen [78]). A similar observation is made with respect split tensile strength of concrete (Fig. 6.5). These observations could be due to use of fly ash as a binder in concrete would result in better packing.

Also, the calcium oxide content in class F fly ash varies from 0.7 to 7.5% by mass where as ultratech ordinary Portland cement is having 61 to 63% by mass in the form of $C_3S$, $C_2S$, $C_3A$ and $C_4AF$ which liberates heat during the hydration process. Hence the replacement of cement with fly ash in concrete will reduce the heat of hydration in concrete which in turn induces increase in strength and setting time of concrete. According to Fraay et al. [51], in case of class F fly ash, the glassy mineral in fly ash is broken down only when the pH value of the pore water is at least about 13.2, and the increase in the alkalinity of the pore water requires that a certain amount of hydration of the Portland cement in the mix has taken place.

Moreover, the reaction products of Portland cement precipitate on the surface of the
fly ash particles, which act as nuclei. When the pH of the pore water becomes high enough, the products of reaction of the fly ash are formed on the fly ash particles and in their vicinity. i.e., calcium silicate hydrates($\text{C-S-H}$), which are less denser when compared to C-S-H formed by hydration of cement, due to this the C-S-H formed by the reaction of fly ash with pore water of hydrated cement fill the voids in concrete. Thus making concrete much denser than with ordinary cement. But as the fly ash content increases more than 15 percent, it dilutes the main contribution of Portland cement Bumrongjaroen and Livingston[58]. The relative importance of these individual effects depends on the chemical composition of the glassy phase of the fly ash.

- In case of DCN specimens, from plots of variation between the applied load corresponding to first crack and the percentage replacement of fly ash, it is observed that the first crack load ($P_{cr}$) varies with ($a/w$) ratio. For a given $a/w$ ratio, the first crack load is maximum at 15% replacement. For a given percentage replacement, as expected, the first crack load, in general, decreases with the increase in ($a/w$) ratio (Fig. 6.6). This is due to decrease in shear resisting area.

- As it can be noted from Fig. 6.3 (final failure crack patterns of DCN specimens) in majority of the cases the cracks are vertical. However, in some of the specimens the cracks show some deviation from the verticality (i.e the crack propagates in an inclined direction from the tip of the notch). This could be due to random inhomogenities in concrete, due to the application of shear force over wide area [13] and/or due to the frictional force generated between loading platens and the specimen [81-83].
From Fig. 6.3, it can be observed that for all percent replacements, for higher values of (a/w), i.e. 0.5 and 0.6 the failure occurs due to the formation of cracks in the ligaments ahead of both the notches; more or less in the vertical direction. In the remaining specimens failure occurs due to propagation of cracks in the ligaments ahead of a single notch. This is also observed in numerical simulations on DEN (double edge notch) and DCN (double centered notch) specimens by Zhu and Tang [5]. A possible reason could be provided using the weakest link hypothesis proposed by Bolotin [85] and Witmann et al. [44]. That is, it is possible that if interfacial transition zone ahead of a notch tip develops into a weakest link and due to stress concentration the crack will propagate in line with the notch in a ragged fashion. This crack would suppress the formation of a crack ahead of the other notch. And thus, Mode - II fracture would occur in these specimens by formation of cracks along one of the two notches. For the specimens with (a/w) = 0.5 and 0.6, cracks develop ahead of both the notches. This could be due to the fact that the shear resisting areas are lesser for compared to specimens with (a/w) = 0.3 and 0.4 and all the four ligaments effectively participate in resisting the shear. Thus, the shear resisting ligaments can be construed to be in parallel in offering resistance to the propagation of crack. This behavior is also witnessed through higher shear strength at first crack of specimens with (a/w) = 0.5 and 0.6 (Fig. 6.7).

From the plots of experimental load – displacement curves (viz. Fig. 6.10.a-d), it has been noted that at a load approximately equal to 50 percent of first visual cracking load the micro-cracking starts resulting in nonlinearity. This observation is consistent with the conclusions drawn by Reinhardt and Xu [30] and by Sih [84].
From the plots of fracture energy, $G_{IF}$ versus percentage replacement with fly ash in place of cement (Fig. 6.12), it is observed that, the fracture energy is maximum for 15% replacement, irrespective of (a/w) ratio. Thus, the fracture energy follows similar trend as that of cube compressive strength and split tensile strength.

As the strains are occurring at $P_{cr}/2$ (Fig. 6.13), this indicates that the internal micro cracking has initiated before the formation of the first visible crack, which is also reflected in load – displacement diagrams (Fig. 6.10.a-d) that non linearity is starting approximately at $P_{cr}/2$.

Based on the available test results on Mode - I fracture of concrete, Bazant and Giraudon [47] had proposed an equation for the calculation of Mode - I fracture energy of concrete (equation 12 of Ref. [47]). Though they have not considered fly ash as a partial replacement for cement in concrete, for the sake of comparison the Mode - I fracture energy ($G_{IF}$) of the concretes considered in the present study are also calculated for different (a/w) ratios and for a particular percentage replacement (Appendix B-8). The average $G_{IF}$ and $G_{II}^e$ for a given percentage replacement (averaged over different a/w ratios considered) are calculated and the ratio between $G_{IF}$ and $G_{II}$ are shown in Table 6.5. The average value of the ratio between $G_{II}^e$ and $G_{IF}$ is obtained as 21.11. The increased fracture energy in Mode - II could be due to extensive aggregate interlock forces that are not present in Mode - I fracture (Swartz et al. [19]).
Table. 6.5 Comparison of $G_{IF}$ and experimental fracture energy $G_{IIF}^e$ of concrete with varying percentages of fly ash replacing ordinary Portland cement in concrete

<table>
<thead>
<tr>
<th>Percentage of fly ash replacing cement</th>
<th>Fracture energy (Mode–I) fracture $G_{IF}$ (N/m)</th>
<th>Experimental mean fracture energy in Mode–II fracture $G_{IIF}^e$ (N/m)</th>
<th>$G_{IIF}^e/G_{IF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>89.33</td>
<td>1540.30</td>
<td>17.24</td>
</tr>
<tr>
<td>5</td>
<td>91.19</td>
<td>1978.10</td>
<td>21.69</td>
</tr>
<tr>
<td>10</td>
<td>93.82</td>
<td>2342.76</td>
<td>24.97</td>
</tr>
<tr>
<td>15</td>
<td>96.89</td>
<td>2507.16</td>
<td>25.88</td>
</tr>
<tr>
<td>20</td>
<td>94.48</td>
<td>2107.03</td>
<td>22.30</td>
</tr>
<tr>
<td>25</td>
<td>92.19</td>
<td>1724.90</td>
<td>18.71</td>
</tr>
<tr>
<td>30</td>
<td>89.98</td>
<td>1530.53</td>
<td>17.01</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>21.11</td>
</tr>
</tbody>
</table>

6.5 Probabilistic analysis on Mode – II fracture energy of concrete with fly ash partially replacing ordinary Portland cement

Though the tests have been conducted under controlled conditions (neglecting the dimensional and load variations) due to inherent randomness in material characteristics, the displacement of
The specimen at any given load level exhibits variations (as indicated earlier) and hence should be considered as a random variable. This is also evident from the experimentally observed variations in fracture energies of two nominally similar samples as reported by Reinhardt and Xu [30]. In order to characterize the variations in fracture energy of a given specimen probabilistic analysis of load-displacement curve (idealized as a tri-linear curve) is carried out using Monte Carlo simulation approach.

From the plots of load – displacement (P - \( \delta \)) curves of DCN specimens (Fig. 6.10.a-d), it is observed that, they can be idealized as tri-linear curves; considering the linearity between zero load to half the first crack load, (P_{cr}/2), half the first crack load to the first crack load (P_{cr}) and from first crack load to the ultimate load (P_u). Fig. 6.14 shows the both the experimental and idealized P - \( \delta \) curves typically for 5 percent replacement and (a/w) = 0.3. The tri - linear idealization seems to be consistent with the results presented by Reinhardt and Xu [30]. The percentage error between tri-linear fit and the polynomial fit has been computed for different combinations of percentage replacements and (a/w). The errors are found to be small suggesting that tri-linear idealization of P - \( \delta \) curve is satisfactory. Typically these values for 10% replacement and (a/w)= 0.4, and, 15% replacement and (a/w) = 0.5 are 3.55% and 2.30%, respectively. Also, the areas under the load – displacement curve between P_{cr} and P_u for the sample combination considered are 79.35% and 79.31%, respectively. This observation suggests that the energy dissipated between P_{cr} and P_u is very significant than that up to P_{cr}. 
Fig. 6.14 Experimental and idealized load-displacement curves at 5% replacement of cement with fly ash and a/w=0.3

From the tri-linear plots, stiffnesses corresponding three linear portions i.e., $K_1$, $K_2$ and $K_3$, and, regression constants i.e., $C_2$ and $C_3$ are evaluated for different combinations of (a/w) and percentage replacements considered. In the probabilistic analysis, first crack and ultimate loads and, $K_1$, $K_2$ and $K_3$ are considered as random variables. The values of these variables, estimated from the experimental results are considered as mean values. Due to the paucity of the relevant experimental data on fly ash in concrete the COV (Coefficient Of Variation) values of these random variables are assumed. The COV values assumed for $P_{cr}$, $P_u$, $K_1$, $K_2$ and $K_3$ are 0.05, 0.10, 0.15, 0.15 and 0.10 respectively. These values are assumed based on the experimental results on three nominally similar specimens tested (Appendix D-5). Also, it is noted that the aim of the present investigation is to examine whether the values of COV of these variables capture the experimentally observed variations in load-displacement behavior for a given percentage replacement irrespective of (a/w) so that a characteristic fracture energy can be defined for a
given percentage replacement. All the variables are assumed to follow lognormal distribution. It is noted that lognormal distribution is generally used to describe the material inhomogeneity (Paramanova et al. [86], Chen et al. [87]). Ten thousand simulations cycles are used. Using ten thousand simulation cycles with 95% confidence the population mean would be contained within an interval of 0.0392S (where S is the sample standard deviation) around the sample mean (Ang and Tang[88]). Similarly, 95% upper confidence limit for population variance is obtained as 0.977 S². A small computer program is written to carry out Monte Carlo simulation of load-displacement behavior and calculation of fracture energy. The simulation consists of the following steps:

1. Generation of ten thousand samples of the basic random variables considered namely, \( P_{cr} \), \( P_u \), \( K_1 \), \( K_2 \) and \( K_3 \) for a given combination of percentage replacement and (a/w).

2. Considering the deformations at loads \( P_{cr}/2 \), \( P_{cr} \) and \( P_u \) as \( d_1 \), \( d_2 \) and \( d_3 \) with \( C_2 \) and \( C_3 \) as the intercepts of the lines with slopes \( K_2 \) and \( K_3 \), for each sample the following procedure is followed to generate the \( P-d \) curve:
   - At the intersection of the lines with slopes \( K_1 \) and \( K_2 \) the load is \( P_{cr}/2 \), the constant \( C_2 \) is evaluated from the expression \( C_2 = (K_1 - K_2) P_{cr}/(2 K_1) \), where \( K_1 = P_{cr}/(2d_1) \).
   - At the intersection of lines with slopes \( K_2 \) and \( K_3 \), the load is \( P_{cr} \) and the constant \( C_3 \) is evaluated from the expression \( C_3 = (K_2 - K_3) d_2 + C_2 \)
     where \( \delta_2 = (P_{cr} - C_2)/K_2 \)
   - For each set of randomly generated values of \( P_{cr} \), \( P_u \), \( K_1 \), \( K_2 \), \( K_3 \) and calculated values of \( C_2 \) and \( C_3 \) a tri-linear load-deformation curve is developed with the following conditions.
     i. The \( P_{cr} \) should be less than \( P_u \).
ii. The slope $K_2$ should be less than the slope $K_1$ (i.e. $d_1$ should be less than $d_2$)

iii. The slope $K_3$ should be less than the slope $K_2$ (i.e. $d_2$ should be less than $d_3$)

iv. The ratios of $d_2/d_1$ and $d_3/d_2$ should be less than 14 (this limit is based on the experimental observations)

3. The area under the generated tri-linear load deformation curve is calculated by summing up the three segmental areas (shown in Fig. 6.14). The corresponding fracture energy is calculated using the area under the load-displacement curve, divided by the shear resisting ligament cross sectional areas.

Using the results of simulation, for each combination considered, the statistical properties of the fracture energy (viz. mean, COV, skewness and kurtosis) are estimated. Fig.6.15.a – 6.15.c Also, the information about the frequency distribution of probabilistic fracture energy $G_{II}$ is obtained for all the percentage replacements and (a/w) ratios considered.

6.6 Results of probabilistic analysis and discussion

The variations in fracture energies estimated using the experimental results are presented in Table 6.6. A similar attempt is made to study the variations based on the results of probabilistic analysis of fracture energy. The results of this study are presented in Table 6.7. The probabilistic mean values have been considered here because in Table 6.6, for a given combination of percentage replacement and (a/w) ratio the average $G_{II}$ of three specimens are considered. Just as in Table 6.6, the minimum, maximum and average values of $G_{II}$ are presented. By comparing the results presented in Tables 6.6 and 6.7, it is clear that the assumed statistical properties (namely mean, coefficient of variation) of variables seem to capture the experimentally observed variations.
Table 6.6 - Minimum, maximum and average fracture energies based on the mean fracture energies obtained from experimental results

<table>
<thead>
<tr>
<th>% replacement of cement with fly ash</th>
<th>G_\text{IF} from experiment (N/m)</th>
<th>Minimum G_\text{IF} (N/m)</th>
<th>Maximum G_\text{IF} (N/m)</th>
<th>Average G^c_\text{IF} (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/w=0.3</td>
<td>1747.82 1434.38 1571.32 1407.69</td>
<td>1407.69</td>
<td>1747.82</td>
<td>1540.30</td>
</tr>
<tr>
<td>a/w=0.4</td>
<td>2159.64 1933.44 2145.93 1673.41</td>
<td>1673.41</td>
<td>2159.64</td>
<td>1978.10</td>
</tr>
<tr>
<td>a/w=0.5</td>
<td>2255.13 2301.25 2549.04 2265.63</td>
<td>2255.13</td>
<td>2549.04</td>
<td>2342.76</td>
</tr>
<tr>
<td>a/w=0.6</td>
<td>2384.11 2464.49 2638.07 2541.97</td>
<td>2384.11</td>
<td>2638.07</td>
<td>2507.16</td>
</tr>
</tbody>
</table>

Table 6.7 - Minimum, maximum and average fracture energies based on the mean fracture energies obtained from probabilistic analysis

<table>
<thead>
<tr>
<th>% replacement of cement with fly ash</th>
<th>G_\text{IF} from probabilistic analysis (N/m)</th>
<th>Minimum G_\text{IF} (N/m)</th>
<th>Maximum G_\text{IF} (N/m)</th>
<th>Average G^p_\text{IF} (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/w=0.3</td>
<td>G_\text{IF} from probabilistic analysis (N/m)</td>
<td>Minimum G_\text{IF} (N/m)</td>
<td>Maximum G_\text{IF} (N/m)</td>
<td>Average G^p_\text{IF} (N/m)</td>
</tr>
<tr>
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<tr>
<td>---</td>
<td>------</td>
<td>------</td>
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<td>------</td>
</tr>
<tr>
<td>0</td>
<td>1735.27</td>
<td>1414.28</td>
<td>1626.50</td>
<td>1440.61</td>
</tr>
<tr>
<td>5</td>
<td>2084.61</td>
<td>1771.40</td>
<td>2119.72</td>
<td>1626.85</td>
</tr>
<tr>
<td>10</td>
<td>2131.49</td>
<td>2160.62</td>
<td>2383.37</td>
<td>2165.36</td>
</tr>
<tr>
<td>15</td>
<td>2330.44</td>
<td>2336.08</td>
<td>2567.17</td>
<td>2505.62</td>
</tr>
<tr>
<td>20</td>
<td>2050.78</td>
<td>2028.64</td>
<td>2274.11</td>
<td>1953.71</td>
</tr>
<tr>
<td>25</td>
<td>1798.24</td>
<td>1727.47</td>
<td>1755.74</td>
<td>1556.69</td>
</tr>
<tr>
<td>30</td>
<td>1617.29</td>
<td>1512.78</td>
<td>1528.75</td>
<td>1324.23</td>
</tr>
</tbody>
</table>

As a next step, it is proposed to examine the actual probability distributions of $G_{HF}$. The values of coefficient of variation, coefficients of skewness and kurtosis (in excess of 3) for various percentages are shown in Figs. 6.15-a to 6.15-c.

The variation of the slopes $K_1$, $K_2$, and $K_3$ of the tri-linear plots of load-displacement relations with different percentage replacements for all (a/w) ratios considered is presented in Figs. 6.16-a to 6.16-c.
From the figures 6.15-a, 6.15-b and 6.15-c the following points are observed.

1. From the plots of tri-linear load-displacement curves of various percentage replacements and (a/w) ratios, it is found that the area under the curves between $P_{cr}$ and $P_u$ is significant and this
area contributes maximum to the $G_{II_F}$. Thus, the trend and variations in $K_3$ are going to significantly affect the variations in $G_{II_F}$. It is noted from the graph between $K_3$ and percentage replacement (Fig. 6.16-c) the slope $K_3$ for replacements other than zero are lower than zero percent replacement, hence greater variations in area under load displacement curve causes more variations in $G_{II_F}$, hence more value of COV for $G_{II_F}$ for percentages other than zero percent replacement.

2. It is noted that for combinations of $(a/w)$ and percentage replacements considered, the skewness coefficients are positive, indicating that the $G_{II_F}$ distribution would have longer falling tails compared to rising tails. The skewness coefficients of $G_{II_F}$ (Fig. 6.15-b) is more for percentage replacements of 10 and 15 percent, where as they remain almost constant at other percent replacements. This observation may be due to getting higher $G_{II_F}$ values for 10 and 15 percent replacements. (Table. 6.7)

3. Since the actual values of coefficient of kurtosis (in excess of 3) are positive and greater than zero, it should be expected that the distribution of $G_{II_F}$ would be more peaked than a normal distribution fitted for the same mean and standard deviation (as obtained from simulation).

4. The kurtosis coefficients of $G_{II_F}$ distributions remains constant for the percentage replacements considered irrespective of $(a/w)$ (except of 10 percent replacement with $(a/w)$=0.4 and 15 percent replacements with $(a/w)$ =0.3). This observation suggests that the degree of peakness of the distribution of $G_{II_F}$ remains almost constant.
5. In order to examine the observations made in 2 to 4 above, further the histograms of the probabilistic fracture energies, $G_{IF}$ are plotted for different (a/w) ratios, for various percentage replacements considered. The same are shown in Figs. 6.17.a – 6.17.g for 0, 5, 10, 15, 20, 25 and
30 percentage replacements. In these plots, the probability densities obtained from simulation and those fitted to normal and lognormal distributions superposed. The number of bins chosen are based on Sturges formula (that is, number of intervals = 1 + 3.3 log (number of simulation cycles)). [Benjamin and Cornell][89]  It is noted that the distribution of $G_{IIF}$ is positively skewed and is more peaked compared to normal distribution.

6. An attempt has been to check the goodness-of-fit of normal- and lognormal- distributions for the observed (i.e. the one obtained from Monte Carlo simulation) $G_{IIF}$ frequency distribution. K-S (Kolmogorov – Smirnov) test is used for this purpose. The results of K-S test are presented in Table 6.8. From these results, for different combinations of $(a/w)$ and percentage replacements considered, it is found that lognormal distribution cannot be rejected at 5 percent significance level. This suggests that the variations in $G_{IIF}$ can be described using lognormal distribution. Using the information that the $G_{IIF}$ follows lognormal distribution and by defining the characteristic fracture energy corresponds to 5% fractile of the distribution, the same can be obtained from

$$G^*_{IIF} = \frac{G_{IIF}}{\sqrt{1+C\sigma^2}} e^{-1.65\sqrt{\ln(1+C\sigma^2)}} \quad \Rightarrow (6.1) \quad (Appendix B-9)$$
Table 6.8  K-S test statistic of $G_{III}$ assuming it to follow lognormal distribution

<table>
<thead>
<tr>
<th>Percentage replacement of cement with fly ash</th>
<th>K-S test statistic to be compared with the critical value*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a/w=0.3</td>
</tr>
<tr>
<td>0</td>
<td>0.004809</td>
</tr>
<tr>
<td>5</td>
<td>0.012802</td>
</tr>
<tr>
<td>10</td>
<td>0.017876</td>
</tr>
<tr>
<td>15</td>
<td>0.010706</td>
</tr>
<tr>
<td>20</td>
<td>0.014416</td>
</tr>
<tr>
<td>25</td>
<td>0.012228</td>
</tr>
<tr>
<td>30</td>
<td>0.010039</td>
</tr>
</tbody>
</table>

*The critical value of K-S test statistic, at 5% significance level, is 0.0136.
Fig. 6.17.a Frequency distributions of fracture energy $G_{II}$ in Mode – II fracture of concrete with zero percent replacement of cement with fly ash for $a/w=0.3$, $0.4$, $0.5$, and $0.6$. The observed frequency is compared with normal and lognormal distributions.
5% replacement with fly ash and a/w=0.3

5% replacement with fly ash and a/w=0.4

5% replacement with fly ash and a/w=0.5

5% replacement with fly ash and a/w=0.6

Fig. 6.17.b Frequency distributions of fracture energy $G_{II}$ in Mode – II fracture of concrete with 5 percent replacement of cement with fly ash
Fig. 6.17.c Frequency distributions of fracture energy $G_{II}$ in Mode – II fracture of concrete with 10 percent replacement of cement with fly ash
Fig. 6.17.d Frequency distributions of fracture energy $G_{II}$ in Mode – II fracture of concrete with 15 percent replacement of cement with fly ash
Fig. 6.17.e Frequency distributions of fracture energy $G_{II}$ in Mode – II fracture of concrete with 20 percent replacement of cement with fly ash.
Fig. 6.17.f Frequency distributions of fracture energy $G_{II}$ in Mode – II fracture of concrete with 25 percent replacement of cement with fly ash
Fig. 6.17.g Frequency distributions of fracture energy $G_{II}$ in Mode – II fracture of concrete with 30 percent replacement of cement with fly ash
7. From the variations of probabilistic $G_{\text{II}}$ with respect to (a/w) for a given percentage replacement (Table. 6.7), it has been noted that probabilistic $G_{\text{II}}$ does not vary significantly. Hence, average value of $G_{\text{II}}$ is considered to be representative for a given percentage replacement. This observation would not only help in simplifying analysis but also helps in fitting a polynomial for variation of $G_{\text{II}}$ with percentage replacement. A second degree polynomial is fitted to the probabilistic mean $G_{\text{II}}^p$ (which is in good agreement with experimental mean values) variation with percentage replacement ($p_r$) as shown in Fig. 6.18, and the same is given by

$$
\tilde{G}_{\text{II}} = -3.50 \, p_r^2 + 100.08 \, p_r + 1548.18 \quad \rightarrow (6.2)
$$

Fig.6.18 Goodness of fit curve for probabilistic mean values of $G_{\text{II}}$

Where $\tilde{G}_{\text{II}}$ is the mean Mode - II fracture energy in N/m, $p_r$ is the percentage replacement of ordinary Portland cement with fly ash in the binder content. The coefficient of determination of the above equation, $R^2$, is 0.9022. From Fig. 6.15-a maximum value of COV of $G_{\text{II}}^p$ can approximately be taken as 0.40. Using the mean fracture energy (Eq.6.2) and a COV of 0.40 the characteristic fracture energy can be calculated for different percentage replacements. By
comparing the characteristic values computed using Eqs. (6.1) and (6.2) with the experimental minimum fracture energies for different replacements, it is found that this value encloses the experimental minimum fracture energy (Fig.6.19)

Fig. 6.19 Variation of characteristic fracture energy, experimental minimum fracture energy with variation in percentage of fly ash replacing ordinary Portland cement in concrete

6.7 Study of the ratio between Mode - II and Mode - I fracture energies

Based on the available test results on Mode - I fracture of concrete, Bazant and Giraudon [47] have proposed, from statistical analysis of test data on Mode - I fracture energy, an equation for the prediction of Mode - I fracture energy ($G_{II}$) of concrete (equation 12 of Ref. [47]). Also, it has been pointed out that the fracture energy in Mode - II can be approximately taken as twenty four times that of Mode - I fracture energy. Though they have not considered fly ash concrete, for the sake of comparison the Mode - I fracture energy ($G_{II}$) of the concretes considered in the present study are also calculated for a given percentage replacement (Appendix B-8). The experimental mean $G_{II}$ (i.e.,$G_{II}^c$) and probabilistic mean $G_{II}$ (i.e.,$G_{II}^p$) for a given percentage
replacement (averaged over different a/w ratios considered) are calculated and the ratios between $G_{II}$ and mean $G_{II}$ are presented in Table 6.9.

Table 6.9 – Ratio between the $G_{II}^e$ to $G_{II}$ and $G_{II}^p$ to $G_{II}$ of concrete having different percentage replacements of cement with fly ash

<table>
<thead>
<tr>
<th>Percentage replacement of cement with fly ash</th>
<th>Fracture energy (mode–I) $G_{II}$ (N/m)</th>
<th>Experimental Fracture energy (mode–II) $G^e_{II}$ (N/m)</th>
<th>Probabilistic Fracture energy (mode–II) $G^p_{II}$ (N/m)</th>
<th>$G_{II}^e/G_{II}$</th>
<th>$G_{II}^p/G_{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>89.33</td>
<td>1540.30</td>
<td>1554.16</td>
<td>17.24</td>
<td>17.40</td>
</tr>
<tr>
<td>5</td>
<td>91.19</td>
<td>1978.10</td>
<td>1900.65</td>
<td>21.69</td>
<td>20.84</td>
</tr>
<tr>
<td>10</td>
<td>93.82</td>
<td>2342.76</td>
<td>2210.21</td>
<td>24.97</td>
<td>23.56</td>
</tr>
<tr>
<td>15</td>
<td>96.89</td>
<td>2507.16</td>
<td>2434.83</td>
<td>25.88</td>
<td>25.13</td>
</tr>
<tr>
<td>20</td>
<td>94.48</td>
<td>2107.03</td>
<td>2076.81</td>
<td>22.30</td>
<td>21.98</td>
</tr>
<tr>
<td>25</td>
<td>92.19</td>
<td>1724.90</td>
<td>1709.54</td>
<td>18.71</td>
<td>18.54</td>
</tr>
<tr>
<td>30</td>
<td>89.98</td>
<td>1530.53</td>
<td>1495.76</td>
<td>17.01</td>
<td>16.62</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>21.11</td>
<td>20.58</td>
</tr>
</tbody>
</table>

The average value of the ratio between $G_{II}^e$ and $G_{II}$ is obtained as 21.11, and between $G_{II}^p$ and $G_{II}$ is 20.58. A good agreement between these two ratios is expected since, as has been noted, there is a good agreement between the results of experimental and probabilistic values of $G_{II}$ (Tables 6.6 and 6.7). Also, as already reported in the literature, $G_{II}$ values are more than $G_{II}$. The increased fracture energy in Mode - II could be due to extensive aggregate interlock forces that are not present in Mode - I fracture (Swartz et al [19]). This clearly indicates the
chosen DCN specimen geometry fails in predominant Mode – II fracture. It also is noted that both the ratios are slightly less than 24 (as reported in [47]) for normal concrete. However, it may be noted that the authors [47] have obtained a COV of 0.299 for the Mode – I fracture energy indicating that there can be large variations in fracture energy. However, according to Reinhardt and Xu [30] this ratio can be between 20 to 25 for normal strength concrete. Reinhardt and Xu [30] based on the tests on double edge notched specimens they were able to trigger shear cracks. The average values of the ratios obtained for concrete with fly ash are nearer to lower bound values reported by Reinhardt and Xu [30].