CHAPTER - 5

EXPERIMENTAL INVESTIGATIONS AND PROBABILISTIC ANALYSIS ON
MODE – II FRACTURE OF CONCRETE WITH HEMATITE AS COARSE
AGGREGATE PARTIALLY / COMPLETELY REPLACING THE GRANITE
AGGREGATE

5.1 Introduction

Generally the concrete structure which are designed to carry heavy loads within the safe permissible stress limits require larger size of structural elements which involves larger volumes of concrete, the recent advancements in the concrete industry have led to develop high performance concrete mixes to suite the specific needs of structural requirements. Generally these high performance concrete requires higher grades of concrete mix with lower water cement ratio which may have high bond strength between cement paste and aggregates, in such a situation, the strength of coarse aggregate plays a dominant role in improving the strength of the concrete mix.

Naturally occurring metallic hematite aggregate is physically strong, dense, durable and impact resistant than the conventional coarse granite aggregate, which is generally used in case of structures called upon to carry high compressive, impact and shear loads (viz. foundations of massive structures, flat slabs, runway pavements, containment structures etc.). Though little amount of literature available [48,50] related such type of concrete, it is concentrated basically on mechanical and Mode – I fracture characteristics of such type of concrete.
Hence in this thesis, an experimentation is made to study the Mode –II fracture characteristics concrete made with partial/complete replacement of conventional coarse aggregate metal granite with hematite. The conventional coarse aggregate in concrete i.e., granite is replaced with hematite aggregate in different percentages i.e., 0, 25, 50, 75 and 100 by mass and its Mode – II fracture characteristics along with mechanical properties, such as compressive strength and split tensile strength are studied. For conducting in-plane shear (Mode-II fracture) tests, the DCN specimen geometry (Fig.1.4) proposed by Prakash Desayi et al. [1] and Bhaskar Desai [2] is considered with various notch to depth ratios of 0.3, 0.4, 0.5 and 0.6 for each percentage variation in the coarse aggregate content. From the test results fracture energy computations are made and a probabilistic analysis has been also carried out to account for the material inhomogeneity and micro structural randomness, which cause variations in failure loads and corresponding displacements. An attempt is also made to analyze its Mode – II fracture characteristics probabilistically using Montecarlo simulation approach.

The organization of this chapter is arranged as follows. The details of experimental work and experimental observations are presented in section 5.2, the results and discussions of experimental investigations are presented in section 5.3, probabilistic analysis of Mode – II fracture of hematite coarse aggregate concrete are presented in section 5.4, results and discussions of the probabilistic analysis are presented in section 5.5, study of the ratio between Mode- II and Mode – I fracture energies (for both experimental and probabilistic analysis) are presented in section 5.6, Summary and conclusions of the experimental and probabilistic analysis are given in chapter 8, section 8.2.

5.2 Experimental investigations

5.2.1 Materials used
The materials used in the experimental programme (Fig. 5.1) are as follows.

![Materials](image.png)

Fig. 5.1 Super plasticizer, Cement, Sand, Granite coarse aggregate, Hematite Coarse aggregate

**Cement:**

Ordinary Portland cement of 53 grade (I.S:12269-1987)[79] of ultratech brand has been used.

The test results concluded on cement are as follows.

- Initial setting time = 35 minutes
- Final setting time = 500 minutes
- Specific gravity = 3.2
- Fineness modulus = 3 percent
- Normal consistency = 33.5 percent

In order to avoid the possible variation in the properties of cement from various batches all the specimens are prepared from the same batch of cement.

**Water:**

The water used in this experimental investigation is locally available potable water.

**Super plasticizer:**

The super plasticizer used in this experimental investigation is Conplast – SP 430.

**Fine aggregate:**
Locally available chitravathi river sand has been used as a fine aggregate, which is free from clay, silt and organic impurities and passing through 4.75 mm size I.S sieve. Its specific gravity is found to be 2.625. From the sieve analysis results of sand, it is observed that sand confirms to zone – I. The grading curve of sand is shown in Fig.5.2

**Coarse aggregate:**

The coarse aggregate consists of locally available crushed granite metal. This coarse aggregate is replaced with hematite metal as coarse aggregate in 0, 25, 50, 75 and 100 percentages by mass, in concrete. The crushed metallic hematite aggregate, is procured from Bellary Iron ore Private limited, Bellary, Karnataka, both the aggregates are passing through 20 mm I.S sieve.

Table 5.1 Physical properties of granite and hematite aggregates

<table>
<thead>
<tr>
<th>Physical property</th>
<th>Granite</th>
<th>Hematite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose density</td>
<td>1380 Kg/m³</td>
<td>2404.67 Kg/m³</td>
</tr>
<tr>
<td>Compacted density</td>
<td>1575 Kg/m³</td>
<td>2713.00 Kg/m³</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>2.625</td>
<td>4.837</td>
</tr>
<tr>
<td>Impact value</td>
<td>9.15%</td>
<td>8.175%</td>
</tr>
</tbody>
</table>
5.2.2 Cleaning of Moulds

Cleaning of the moulds is done as explained section 4.2.2

5.2.3 Mixing of the ingredients

The M20 concrete mix has been designed using ISI method (IS:10262-1982)[80] for zero percent replacement of coarse aggregate. The mix proportion obtained is 1:1.58:2.86 with water cement ratio of 0.5. The design procedure is presented in Appendix-A. Keeping the mass of the coarse aggregates constant, the granite coarse aggregate has been replaced by hematite coarse aggregate in percentages of 0, 25, 50, 75 and 100 by mass. For each percentage replacement of coarse aggregate considered, the materials are mixed in the standard way. That is, at first the fine aggregate and cement are weighed according to their proportion in the concrete mix. Then these materials are mixed thoroughly in dry condition. Then the coarse aggregates i.e., hematite and granite are weighed according to their proportion in the concrete mix and mixed thoroughly in dry condition. Then the dry mixture of cement and fine aggregate is spread uniformly over the
coarse aggregate mixture of hematite and granite and thoroughly mixed in dry condition. Then the measured quantity of water with water cement ratio of 0.5 is added to this dry mix and then mixed thoroughly. For each percentage replacement of fine aggregate considered, to have a consistent workability of the mix, a slump of 100±10 mm is maintained. For this, trial mixes are made in arriving the required slump with the help of superplasticizer, conplast SP – 430 at constant water cement ratio of 0.5. This dosage of super plasticizer is then added in casting the specimens of a given batch.

5.2.4 Casting of the specimens

For each percentage replacement in coarse aggregate considered, 12 cubes of DCN specimens of size of 150x150x150 mm with notch depth ratios, (a/w), of 0.3,0.4,0.5 and 0.6; three plain cubes of size 150x150x150 mm and, three cylinders of size 150 mm diameter and 300 mm height are cast. For each combination of parameters considered in this investigation three similar DCN specimens have been cast and tested. Thus, a total number of 90 specimens are cast in this experimentation (zero percent replacement being common for all other experiments). The geometrical details of the specimen are shown Fig.1.4 and explained in section 4.2.4.

For all test specimens, moulds are kept on the vibrating table and the concrete is poured into the moulds in three layers each layer being compacted thoroughly and uniformly with a tamping rod to avoid honey combing. Finally all specimens are vibrated on the table vibrator after filling up the moulds up to the brim. The vibration is effected for seven seconds and it is maintained constant for all specimens and all the other castings. The steel plates forming notches are removed after three hours of casting carefully and the specimens are neatly finished. However the specimens are demoulded after twenty four hours of casting and are kept immersed in a clean
water tank for curing (Fig.4.3). After twenty eight days of curing the specimens are taken out of water and are allowed to dry under shade for few hours.

5.2.5 Loading arrangement and Testing of the specimens

All the specimens are white washed and tested in a 2000 kN digital compression testing machine with a uniform rate of loading of 0.1 kN/sec. The loading arrangement for the specimens considered in this experimentation i.e., for cubes, cylinders and DCN specimens are similar to that as considered in section 4.2.5 (Figs.4.4, 4.5, and 4.6).

The companion specimens of cubes are tested for their compressive strength, cylinders are tested for the split tensile strength and DCN specimens are tested for characterizing their in-plane shear strength. It is to be noted that, though the loading is applied continuously during testing of the DCN specimens, the in-plane displacements of central one third portion if any, are noted at regular loading increments of 10 kN and the initiation and propagation of cracks is observed continuously. The overall test setup for DCN specimens is similar to as shown in Fig.4.6. For measuring the in-plane shear displacement of the central one third portion of the DCN specimen, four LVDT’s (with least count of 0.01 mm) are arranged as shown in Fig.4.6. The LVDT’s have been arranged such that they represent the localized in-plane shear displacement, there by avoiding noise (Reinhardt and Xu) [30]. To prevent horizontal cracking vertical steel clamps are fixed to the outer ends of the DCN specimen. The LVDT readings are taken at load increments of 10 kN. Then, the average of four LVDT readings is taken to represent the in-plane shear displacement at a given stage of loading. Also, the loads corresponding to the first visual crack initiation load at the end of the notch tips (Pcr) and the ultimate load (Pu) are noted. However, for plotting the load-displacement curve, for specified (a/w) ratio and percentage
replacement, the displacement, at a given load level is again averaged over three specimens, so that the most best average load-displacement curve is obtained.

During the experimentation, while in most of the cases the crack initiation is visually observed at the top edge of one or both the notches and then the cracks formed at the bottom edge of the one or both the notches, in some cases visual crack initiation (first) occurred at the bottom edge of the notch(es) and then at the top edge of the notch(es). This observation could be due to inherent inhomogeneity of concrete. In some of the specimens the failure has occurred due to propagation of crack in the ligaments ahead of the same notch. Thus, formation of a single almost co-planar (though slightly jaggered) crack has led to failure in these specimens. A possible explanation for this observation is hypothesized in the earlier chapter 4 based on weakest link hypothesis(section 4.3-4).

Final crack patterns are shown in Fig. 5.3. It can be observed from Fig. 5.3 that there exists a deviation from verticality of the crack path (i.e the crack propagates in an inclined direction from the tip of the notch). The experimental observations of van Mier [81,82] and van Geel [83] on concrete prisms, for both uniaxial and confined compression, have shown that the frictional resistance from the loading system will have effect on the direction of the shear crack orientation. Also, Bazant and Pfieffer [13] showed that crack band propagates sideways when shear force zone is wide and vertically when the shear force zone is narrow (it is noted that the shear force applied in the present experimental programme is over wider area and hence inclination of cracks). From the experimental final crack patterns it is found that, the cracks ahead of notch tips are comparatively more inclined in specimens with lesser (a/w) ratios i.e., a/w=0.3 and 0.4 than those with higher (a/w) ratios i.e., 0.5 and 0.6 considered.
Fig. 5.3 Final crack patterns of cube and DCN specimens having different percentage replacements with hematite as coarse aggregate

For a given percentage replacement with hematite coarse aggregate in concrete, the average values of compressive strength of cube specimens and split tensile strength of cylinder specimens are calculated. The Fig.5.4 shows the variations in ultimate strength on cubes in direct compression with variation in percentage replacement of coarse aggregate content. From this it
can be observed that the cube compressive strength gradually increases with increase in the hematite aggregate content up to 100 percent replacement.

![Graph showing the variations in cube compressive strength](image)

\[ Y = 26.8 + 0.1647X - 0.00439X^2 + 5.456E-5X^3 - 1.99467E-7X^4 \]

\[ R^2 = 1.00 \]

Fig. 5.4 Variation in the cube compression strength at different percentage replacements with hematite as coarse aggregate

The Fig. 5.5 shows the variations in split tensile strength of cylinders with variation in percentage replacement of coarse aggregate content. From this it can be observed that, the split tensile strength also increases with the increase in the hematite aggregate content in concrete.
Fig. 5.5 Variation in the Split tensile strength at different percentage replacements with hematite as coarse aggregate

It is noted that, as reported by Keru Wu et. al [48], the replacement of granite coarse aggregate with hematite coarse aggregate, would increase the compressive strength and split tensile strength of concrete with increase in the hematite content.

For DCN specimens, for each (a/w) ratio and percentage replacement considered, the average crack initiation load ($P_{cr}$), that is first crack load, and ultimate load ($P_u$) are noted. The variation of first visual cracking load $P_{cr}$ for different combinations of (a/w) with variations in the percentage replacement of hematite aggregate in place of granite as coarse aggregate is shown in Fig.5.6. From this figure it is observed that, the first crack load increases with increase in hematite aggregate replacement content for each of the (a/w) ratio considered. The shear strength corresponding to crack initiation ($P_{cr}$) for different combinations of (a/w) and percentage replacements are shown in Fig. 5.7. From this figure it is observed that, with the increase in percentage replacement with hematite aggregate content, for each (a/w) considered, the
corresponding shear strength at crack initiation load ($P_{cr}$) is also increased. The variation of ultimate load, $P_u$ for different combinations of (a/w) and percentage replacements are shown in Fig. 5.8. From this figure the ultimate load, $P_u$ causing failure of the DCN specimens, also increases with increase in hematite aggregate replacement content for each (a/w) ratio considered.

![Graph showing variation of first crack load $P_{cr}$ at different percentage replacements with hematite as coarse aggregate](image)

Fig. 5.6 Variation of First crack load $P_{cr}$ at different percentage replacements with hematite as coarse aggregate
Fig. 5.7 Variation of shear stress at first crack at different percentage replacements with hematite as coarse aggregate

Fig. 5.8 Variation of Ultimate load $P_u$ at different percentage replacements with hematite as coarse aggregate
5.2.6 Load verses in-plane shear displacement behavior

An attempt has been made in this investigation to study the variations in Mode - II fracture energy of concrete with varying percentages of replacement of granite with hematite as coarse aggregate. In order to carry out this study, experimentally determined load-displacement curves of DCN specimens are used. As already indicated, for a given combination of (a/w) ratio and percentage replacement, there are three nominally similar specimens. At every 10 kN load increment, the mean value of displacement of the three specimens is computed. Also the average visual crack initiation load ($P_{cr}$) and ultimate load ($P_u$) (of three nominally similar specimens) are considered for drawing the load - displacement curve. Thus, for a given combination of parameters considered (for a given notch to depth ratio and percentage replacement of coarse aggregate) one load- displacement curve is obtained. A typical load – displacement curve is shown in Fig.5.9 for 50 percent replacement of granite with hematite coarse aggregate and (a/w) ratio=0.3

![Graph showing load vs. displacement relationship]

Fig. 5.9 Variation between load and displacement at 50 percent replacement with hematite as coarse aggregate and (a/w) =0.3
Some other typical load-displacement curves obtained in this investigation are presented in Fig. 5.10. From load-displacement curves it is observed that due to high initial rigidity, the specimens are taking the initial load without undergoing initial deformation. However, in the computation of fracture energy, the load–displacement curve is considered to pass through the origin at zero load. Also, as has been reported in Reinhardt and Xu[30] and by Sih [84], the micro-cracking starts at a load approximately equal to the fifty percent of the first visual cracking load. This is evident from the nonlinearity in the load-displacement curves (Fig. 5.10) around this load.
| a | Load – displacement curves with \((a/w) = 0.3\) at different percentage replacements with hematite coarse aggregate |
| b | Load – displacement curves with \((a/w) = 0.4\) at different percentage replacements with hematite coarse aggregate |
| c | Load – displacement curves with \((a/w) = 0.5\) at different percentage replacements with hematite coarse aggregate |
| d | Load – displacement curves with \((a/w) = 0.6\) at different percentage replacements with hematite coarse aggregate |

Fig. 5.10 Load - displacement curves at 0, 25, 50, 75 and 100 percent replacements of granite coarse aggregate with hematite coarse aggregate and \((a/w)\) ratios of 0.3, 0.4, 0.5 and 0.6

5.2.7 Determination of experimental fracture energy
As defined earlier in section 4.2.7, the area under the load-displacement curve divided by the shear resisting cross section area of the ligaments above and below the notches gives the experimental fracture energy in Mode – II (G_{II}) fracture of DCN specimens. Thus, in the present investigation, the Mode - II fracture energy is obtained by evaluating the area under the load-displacement curve (hatched area shown in Fig.5.9) divided by the corresponding shear resisting area (cross section area of ligaments).

A best fit polynomial equation is fitted to the experimentally obtained load-displacement curve. The area under this curve is computed using ORIGIN software.

From the plot of experimental fracture energy (G_{II}) verses (a/w) ratio (Fig. 5.11), for each percentage replacement of the coarse (granite) aggregate content, it is observed that there is no significant variation in the fracture energy (G_{II}) with variation in (a/w) ratio for a given percentage replacement of coarse aggregate. A similar observation was made by Reinhardt and Xu [30] on edge notched specimens with different ligament lengths (based on this observation, they reported average fracture energy G_{II} as a representative quantity). Therefore, the average fracture energy of DCN specimens, for a given percentage replacement in the coarse aggregate content, corresponds to a specimen with average shear resisting area (i.e., 24750 mm^2). Is also noted that for all (a/w) ratios considered, the fracture energy increases with the increase in percentage replacement of coarse aggregate (Fig. 5.12) A similar observation was made by Keru Wu et al. [48] with respect to fracture energy of hematite metallic aggregate concrete tested in Mode-I.
Fig. 5.11 Variation of experimental fracture energy (G_{III}) with respect to (a/w) ratio at different percentage replacement with hematite coarse aggregate.

Fig. 5.12 Variation of experimental fracture energy (G_{III}) at different percentage replacement with hematite coarse aggregate.
5.2.8 Computed and measured shear strains at $P_{cr}/2$

For average stress – strain behavior of concrete (made with partial/complete replacement of granite coarse aggregate with hematite coarse aggregate), ahead of crack tip of DCN specimens, two dimensional pure shear condition is assumed to estimate the shearing strain. The load – displacements of DCN specimens are measured using the LVDT’s over a gauge length of 100 mm.

A graph is plotted from the data of computed and measured strains at $P_{cr}/2$ (Fig. 5.13), for all percentage replacements and (a/w) ratios considered. The sample calculations involved are presented in Appendix-C.1, typically for the case of a DCN specimen with (a/w) = 0.3 and percentage replacement equal to 50. A line of equality is also shown in Fig. 5.13. From this figure it is observed that, measured strains are slightly higher values than those of computed strains.

![Graph showing comparison between computed and measured shear strains at $P_{cr}/2$](image)

Fig.5.13 Comparison between computed and measured shear strains at an applied load level of $P_{cr}/2$
5.3 Results of experimental observations and discussions

1. From the plot of variation of cube compressive strength ($f_{cu}$) with percentage replacement of granite coarse aggregate with hematite aggregate (Fig.5.4), it is observed that the cube compressive strength increases with increase in hematite coarse aggregate content. This could be due intrinsic strength of hematite aggregate, absorbing loads in concrete. A similar observation was made by Keru Wu et al. [48]

2. From the plot of variation in the split tensile strength with variation in percentage of hematite content, (Fig.5.5), it can be seen that, as the hematite aggregate content in concrete increases in place of granite aggregate, the split tensile strength also increases. This could be due better bonding of cementitious material with the hematite aggregate surface. The experiments of Keru Wu et al. [48] also showed that the split tensile strength increased with the increase in hematite aggregate content in concrete.

3. The experimental final crack pattern of DCN specimens (Fig. 5.3) showed a deviation from verticality of the crack path (i.e., the crack propagates in an inclined direction from the tip of the notch). i.e., in most of the specimens cracks are initiated at the top end of notch tips and then propagated towards the top load simultaneously forming the cracks at the bottom of the notches and propagating towards the bottom reactions with inclination. This could be due to the action of shear force over a wider area [13]. This crack pattern could be, also as inferred by van Meir [81,82] and van Geel [83] due to the development of coulomb friction force developed over the contact area of load on the specimen. The degree of inclination of crack decreases with the increase in notch depth ratio and the angle of inclination is not significantly affected by the change in percentage replacement.

In case of some of DCN specimens failure has occurred along a single notch, which could
be due to differential stress concentrations occurred at notch tips due to random inhomogenities in concrete, leading to from failure along a single notch plane.

4. From the plots of experimental load – displacement curves (Fig. 5.10) it has been noted that at a load approximately equal to 50% of first crack initiating load ($P_{cr}$) the micro-cracking starts resulting in non linearity. This observation is consistent with the conclusion drawn by Reinhardt and Xu [30] and by Sih [84].

5. In case of DCN specimens for all ratios of (a/w), the values of first crack initiation load ($P_{cr}$), shear stress at crack initiation load ($P_{cr}$), and ultimate load ($P_{u}$) increase with the increase in the hematite aggregate content. This could be due to better energy absorption capacity of hematite aggregate content in concrete and better bonding effect of cementitious material with hematite aggregate.

6. From the analysis of experimental fracture energy, $G_{IF}$, obtained using experimental load – displacement curves, it has been found that there is no significant variation in $G_{IF}$ with variation in (a/w) for a given percentage replacement of coarse aggregate (Fig.5.11). A similar observation was made by Reinhardt and Xu [30] on edge notched specimens with different ligament lengths (based on this observation, they reported average fracture energy $G_{IF}$ as a representative quantity).

7. From the Fig.5.13, it is noted that the measured strains are slightly higher than the computed strains (based on equivalent cylinder strength in compression), which could be due difference in behavior of hematite coarse aggregate in direct compression when compared to its shear behavior.

Based on the available test results on Mode - I fracture of concrete, Bazant and Giraudon [47] had proposed an equation for the calculation of Mode - I fracture energy of concrete (equation 12
of Ref. [47]). Though they have not considered hematite as coarse aggregate concrete, for the sake of comparison the Mode - I fracture energy ($G_{IF}$) of the concrete (Appendix B-8) considered in the present study are also calculated for different (a/w) ratios and for a particular percentage replacement. The average $G_{IF}$ and $G_{eII}$ for a given percentage replacement (averaged over different a/w ratios considered) are calculated and the ratio between $G_{IF}$ and $G_{eII}$ are shown in Table 5.2. The average value of the ratio between $G_{eII}$ and $G_{IF}$ is obtained as 24.71. The increased fracture energy in Mode - II could be due to extensive aggregate interlock forces those are not present in Mode - I fracture (Swartz et al [19]).

Table 5.2 Comparison of $G_{IF}$ and experimental fracture energy $G_{eII}$ of concrete with varying percentages of hematite coarse aggregate replacing the granite coarse aggregate

<table>
<thead>
<tr>
<th>Percentage of hematite aggregate replacing granite aggregate</th>
<th>Fracture energy in Mode–I fracture $G_{IF}$ (N/m)</th>
<th>Experimental mean fracture energy in Mode–II fracture $G_{eII}$ (N/m)</th>
<th>$G_{eII}/G_{IF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>89.33</td>
<td>1540.30</td>
<td>17.24</td>
</tr>
<tr>
<td>25</td>
<td>92.56</td>
<td>1755.13</td>
<td>18.96</td>
</tr>
<tr>
<td>50</td>
<td>93.57</td>
<td>2247.72</td>
<td>24.02</td>
</tr>
<tr>
<td>75</td>
<td>95.77</td>
<td>2669.87</td>
<td>27.88</td>
</tr>
<tr>
<td>100</td>
<td>99.68</td>
<td>3534.19</td>
<td>35.46</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>24.71</td>
</tr>
</tbody>
</table>
5.4 Probabilistic analysis on Mode – II fracture energy of concrete with varying percentages of hematite coarse aggregate partially/completely replacing the granite coarse aggregate

Though the tests have been conducted under controlled conditions (neglecting the dimensional and load variations) due to inherent randomness in material characteristics, the displacement of the specimen at any given load level exhibits variations (as indicated earlier) and hence should be considered as a random variable. This was also evident from the experimentally observed variations in fracture energies of two nominally similar samples reported by Reinhardt and Xu [30]. In order to characterize the variations in fracture energy of a given specimen, probabilistic analysis of load-displacement curve (idealized as a tri-linear curve) is carried out using Monte Carlo simulation approach. The same procedure as discussed in the article 4.4 has been adopted for the probabilistic analysis.

From the plots of load – displacement (P - δ) curves of DCN specimens (Fig. 5.10 a -d), it is observed that, they can be idealized as tri-linear curves; considering the linearity between zero load to half the first crack load, (P_{cr}/2), half the first crack load to the first crack load (P_{cr}) and from first crack load to the ultimate load (P_u). Fig. 5.14 shows both the experimental and idealized P - δ curves typically for 25 percent replacement and (a/w) = 0.3. The tri – linear idealization seems to be consistent with the results presented by Reinhardt and Xu [30]. The percentage error between tri-linear fit and the polynomial fit has been computed for different combinations of percentage replacements and (a/w). The errors are found to be small suggesting that tri-linear idealization of P - δ curve is satisfactory. Typically these values for 25% replacement and (a/w)= 0.3, and, 100% replacement and (a/w) = 0.5 are 1.38% and 2.77%, respectively. Also, the areas under the load – displacement curve between P_{cr} and P_u for the
sample combination considered are 82.14% and 69.31%, respectively. This observation suggests that the energy dissipated between $P_{cr}$ and $P_u$ is very significant than that up to $P_{cr}$.

From the tri-linear plots, stiffnesses corresponding three linear portions $K_1$, $K_2$ and $K_3$, and, regression constants $C_2$ and $C_3$ are evaluated for different combinations of ($a/w$) and percentage replacements considered. In the probabilistic analysis, first crack and ultimate load and, $K_1$, $K_2$ and $K_3$ are considered as random variables. The values of these variables, estimated from the experimental results are considered as mean values. Due to the paucity of the relevant experimental data on hematite aggregate in concrete the COV (Coefficient Of Variation) values of these random variables are assumed. The COV values assumed for $P_{cr}$, $P_u$, $K_1$, $K_2$ and $K_3$ are 0.05, 0.10, 0.15, 0.15 and 0.10, respectively. These values are assumed based on the experimental results on three nominally similar specimens tested (appendix C-5). Also, it is noted that the aim of the present investigation is to examine whether the values of COV of these
variables capture the experimentally observed variations in load-displacement behavior for a given percentage replacement irrespective of (a/w) so that a characteristic fracture energy can be defined for a given percentage replacement. All the variables are assumed to follow lognormal distribution. It is noted that lognormal distribution is generally used to describe the material inhomogeneity (Paramanova et al. [86], Chen et al. [87]). Ten thousand simulations cycles are used. Using ten thousand simulation cycles with 95% confidence the population mean would be contained within an interval of 0.0392S (where S is the sample standard deviation) around the sample mean (Ang and Tang[88]). Similarly, 95% upper confidence limit for population variance is obtained as 0.977 S^2. A small computer program was written to carry out Monte Carlo simulation of load-displacement behavior and calculation of fracture energy. The simulation consists of the following steps:

1. Generation of ten thousand samples of the basic random variables considered namely, P_cr, P_u, K_1, and K_2 and K_3 for a given combination of percentage replacement and (a/w).

2. Considering the deformations at loads P_cr/2, P_cr and P_u as d_1, d_2 and d_3 with C_2 and C_3 as the intercepts of the lines with slopes K_2 and K_3, for each sample the following procedure is followed to generate the P-d curve:
   • At the intersection of the lines with slopes K_1 and K_2 the load is P_cr/2, the constant C_2 is evaluated from the expression C_2 = (K_1- K_2) P_cr/(2 K_1), where K_1=P_cr/(2d_1).
   • At the intersection of lines with slopes K_2 and K_3, the load is P_cr and the constant C_3 is evaluated from the expression C_3 = (K_2- K_3) d_2 + C_2 where d_2=(P_cr-C_2)/ K_2.
For each set of randomly generated values of $P_{cr}$, $P_u$, $K_1$, $K_2$, $K_3$ and calculated values of $C_2$ and $C_3$ a tri-linear load-deformation curve is developed with the following conditions.

i. The $P_{cr}$ should be less than $P_u$.  

ii The slope $K_2$ should be less than the slope $K_1$ (i.e. $d_1$ should be less than $d_2$)  

iii The slope $K_3$ should be less than the slope $K_2$ (i.e. $d_2$ should be less than $d_3$)  

iv The ratios of $d_2/d_1$ and $d_3/d_2$ should be less than 9 (this limit is based on the experimental observations)

3. The area under the generated tri-linear load deformation curve is calculated by summing the three segmental areas (shown in Fig. 5.14). The corresponding fracture energy is calculated using the area under the load-displacement curve, divided by the shear resisting ligaments cross sectional area.

Using the results of simulation, for each combination considered, the statistical properties of the fracture energy (viz. mean, COV, skewness and kurtosis) are estimated. (Figs 5.15.a – 5.15.c)

Also, the information about the frequency distribution of fracture energy $G_{II}$ is obtained for all the percentage replacements and (a/w) ratios considered.

**5.5 Results of probabilistic analysis and discussion**

The variations in fracture energies estimated using the experimental results are presented in Table 5.3. A similar attempt is made to study the variations based on the results of probabilistic analysis of fracture energy. The results of this study are presented in Table 5.4. The probabilistic mean values have been considered here because in Table 5.3, for a given
combination of percentage replacement and (a/w) ratio the average $G_{IIF}$ of three specimens are considered. Just as in Table 5.3, the minimum, maximum and average values of probabilistic fracture energy $G_{IIF}$ are presented. By comparing the results presented in Tables 5.3 and 5.4, it is clear that the assumed statistical properties (namely mean, coefficient of variation) of variables seem to capture the experimentally observed variations.

Table 5.3- Minimum, maximum and average fracture energies obtained from experimental results

<table>
<thead>
<tr>
<th>% replacement in coarse aggregate by hematite</th>
<th>Mean $G_{IIF}$ from experimental analysis (N/m)</th>
<th>Minimum of mean $G_{IIF}$ (N/m)</th>
<th>Maximum mean $G_{IIF}$ (N/m)</th>
<th>Average $G_{IIF}$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/w=0.3</td>
<td>1747.82</td>
<td>1407.69</td>
<td>1747.82</td>
<td>1540.30</td>
</tr>
<tr>
<td>a/w=0.4</td>
<td>1434.38</td>
<td>1407.69</td>
<td>1407.69</td>
<td>1407.69</td>
</tr>
<tr>
<td>a/w=0.5</td>
<td>1571.32</td>
<td>1571.32</td>
<td>1571.32</td>
<td>1571.32</td>
</tr>
<tr>
<td>a/w=0.6</td>
<td>1407.69</td>
<td>1407.69</td>
<td>1407.69</td>
<td>1407.69</td>
</tr>
<tr>
<td>0</td>
<td>1747.82</td>
<td>1407.69</td>
<td>1747.82</td>
<td>1540.30</td>
</tr>
<tr>
<td>25</td>
<td>1907.94</td>
<td>1622.22</td>
<td>1907.94</td>
<td>1755.13</td>
</tr>
<tr>
<td>50</td>
<td>2352.38</td>
<td>2166.67</td>
<td>2352.38</td>
<td>2247.72</td>
</tr>
<tr>
<td>75</td>
<td>2892.06</td>
<td>2568.89</td>
<td>2892.06</td>
<td>2669.87</td>
</tr>
<tr>
<td>100</td>
<td>3923.81</td>
<td>3383.33</td>
<td>3929.31</td>
<td>3534.19</td>
</tr>
</tbody>
</table>
Table 5.4- Minimum, maximum and average fracture energies based on the mean fracture energies obtained from probabilistic analysis of $G_{\text{IIc}}$

<table>
<thead>
<tr>
<th>% of replacement in coarse aggregate by hematite</th>
<th>$G_{\text{IIc}}$ from probabilistic analysis (N/m)</th>
<th>Minimum $G_{\text{IIc}}$ (N/m)</th>
<th>Maximum $G_{\text{IIc}}$ (N/m)</th>
<th>Average $G_{\text{IIc}}$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a/w=0.3$</td>
<td>$a/w=0.4$</td>
<td>$a/w=0.5$</td>
<td>$a/w=0.6$</td>
</tr>
<tr>
<td>0</td>
<td>1714.79</td>
<td>1414.28</td>
<td>1626.50</td>
<td>1407.48</td>
</tr>
<tr>
<td>25</td>
<td>1862.69</td>
<td>1770.46</td>
<td>1701.66</td>
<td>1621.38</td>
</tr>
<tr>
<td>50</td>
<td>2281.96</td>
<td>2202.45</td>
<td>2283.76</td>
<td>2094.69</td>
</tr>
<tr>
<td>75</td>
<td>3014.97</td>
<td>2649.98</td>
<td>2663.48</td>
<td>2549.56</td>
</tr>
<tr>
<td>100</td>
<td>3917.08</td>
<td>3254.32</td>
<td>3664.04</td>
<td>3244.47</td>
</tr>
</tbody>
</table>

As a next step, it is proposed to examine the actual probability distributions of $G_{\text{IIc}}$. The values of coefficient of variation, coefficients of skewness and kurtosis (in excess of 3) for various percentages are shown in Figs. 5.15 a – 5.15c.

The variation of the slopes $K_1$, $K_2$, and $K_3$ of the tri-linear plots of load-displacement relations with different percentage replacements for all ($a/w$) ratios considered is presented in Figs. 5.16-a to 5.16-c.
From these figures the following points are noted:

1) The coefficient of variation of $G_{II}$, while in general varies with percentage of replacement for a given (a/w) ratio, it remains almost constant after 50 percent
replacement for (a/w) = 0.4 and 0.5. This could be due to less variation in K_1, K_2 and K_3 values for different (a/w) ratios at different replacement levels. (Fig.5.16.a-5.16.c)

Fig. 5.16.a Variation between K_1 and percentage of replacement with hematite aggregate

Fig. 5.16.b Variation between K_2 and percentage of replacement with hematite aggregate

Fig. 5.16.c Variation between K_3 and percentage of replacement with hematite aggregate

2) A similar observation is made with respect to skewness coefficient. It is noted that for combinations of (a/w) and percentage replacements considered, the skewness coefficients
are positive indicating that the $G_{II}$ distribution would have longer falling tails compared to rising tails.

3) The plot of kurtosis coefficients variation with respect to different percentage replacements with hematite aggregate (Fig.5.15.c) shows that unlike the COV and skewness coefficients, the kurtosis of the $G_{II}$ distribution remains constant for all the percentage replacements considered (except for 0% replacement), irrespective of value of ($a/w$). This observation suggests that the degree of peakness of the $G_{II}$ distribution remains almost constant.

4) Since the actual values of coefficient of kurtosis (in excess of 3) are positive and greater than zero, it should be expected that the distribution of $G_{II}$ would be more peaked than a normal distribution fitted for the same mean and standard deviation (as obtained from simulation).

5) In order to examine the observations made in 2 to 4 above, further the histograms of the probabilistic fracture energies, $G_{II}$ are plotted for different ($a/w$) ratios, for various percentage replacements considered. The same are shown in Figs. 5.17.a – 5.17.e for 0, 25, 50, 75 and 100 percentage replacements. In these plots, the probability densities obtained from simulation and those fitted to normal- and lognormal- distributions are superposed. The number of bins chosen are based on Sturges formula (that is, number of intervals = $1 + 3.3 \log$ (number of simulation cycles)) [Benjamin and Cornell] [89]. It is noted that the distribution of $G_{II}$ is positively skewed and is more peaked compared to normal distribution.

6) An attempt has been made to check the goodness-of-fit of normal- and lognormal-distributions for the observed (i.e. the one obtained from Monte Carlo simulation) $G_{II}$
frequency distribution. K-S (Kolmogorov – Smirnov) test is used for this purpose. The results of K-S test are presented in Table 5.5. From these results, for different combinations of (a/w) and percentage replacements considered, it is found that lognormal distribution cannot be rejected at 5 percent significance level. This suggests that the variations in G_{IF} can be described using lognormal distribution. Using the information that the G_{IF} follows lognormal distribution and by defining the characteristic fracture energy corresponds to 5% fractile of the distribution, the same can be obtained from

\[ G^*_{IF} = \left( \frac{\bar{G}_{II_F} e^{-1.65\sqrt{\ln(1+cov^2)}}}{\sqrt{1 + cov^2}} \right) \rightarrow (5.1) \]  

(Appendix B-9)

Table 5.5 – K- S test statistic of G_{IF} assuming it to follow lognormal distribution

<table>
<thead>
<tr>
<th>% replacement of granite with hematite coarse aggregate</th>
<th>K-S test statistic to be compared with the critical value*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a/w=0.3</td>
</tr>
<tr>
<td>0</td>
<td>0.005148</td>
</tr>
<tr>
<td>25</td>
<td>0.007331</td>
</tr>
<tr>
<td>50</td>
<td>0.010970</td>
</tr>
<tr>
<td>75</td>
<td>0.015905</td>
</tr>
<tr>
<td>100</td>
<td>0.014244</td>
</tr>
</tbody>
</table>

(* Note: The critical K-S value at 5% significance level corresponding to number of degrees of freedom=14 is 0.349)
Zero percent replacement with hematite aggregate and a/w=0.3

Zero percent replacement with hematite aggregate and a/w=0.4

Zero percent replacement with hematite aggregate and a/w=0.5

Zero percent replacement with hematite aggregate and a/w=0.6

Fig. 5.17.a Frequency distributions of fracture energy in Mode-II fracture of concrete with zero percent replacement of granite with hematite coarse aggregate
25 percent replacement with hematite aggregate and a/w=0.3

25 percent replacement with hematite aggregate and a/w=0.4

25 percent replacement with hematite aggregate and a/w=0.5

25 percent replacement with hematite aggregate and a/w=0.6

Fig. 5.17.b Frequency distributions of fracture energy in Mode-II fracture of concrete with 25 percent replacement of granite with hematite coarse aggregate
Fig. 5.17.c Frequency distributions of fracture energy in Mode-II fracture of concrete with 50 percent replacement of granite with hematite coarse aggregate
Fig. 5.17.d Frequency distributions of fracture energy in Mode-II fracture of concrete with 75 percent replacement of granite with hematite coarse aggregate
Fig. 5.17.e Frequency distributions of fracture energy in Mode-II fracture of concrete with 100 percent replacement of granite with hematite coarse aggregate and a/w=0.3, a/w=0.4, a/w=0.5, and a/w=0.6.
7) From the variations of probabilistic $G_{II}$ with respect to (a/w) for a given percentage replacement (Table. 5.4), it has been noted that probabilistic $G_{II}$ does not vary significantly. Hence, average value of probabilistic $G_{II}$ is considered to be representative for a given percentage replacement. This observation would not only help in simplifying analysis but also helps in fitting a polynomial for variation of probabilistic mean $G_{II}^p$ variation with percentage replacement. A second degree polynomial is fitted to the probabilistic mean $G_{II}^p$ (which is in good agreement with experimental mean values) variation with percentage replacement, as shown in Fig.5.18 and the same is given by

$$\bar{G}_{II} = 0.14 p_r^2 + 5.681 p_r + 1535.16 \quad \rightarrow \quad (5.2)$$

![Fig.5.18 Goodness of fit curve for probabilistic mean values of $G_{II}$](image)

Where $\bar{G}_{II}$ is the mean Mode-II fracture energy in N/m, $p_r$ is the percentage replacement of granite with hematite. The coefficient of determination of the above equation, $R^2$ is 0.998. From Fig. 5.15-a, maximum value of COV of $G_{II}$ can approximately be taken as 0.32. Using the mean (Eq. 5.2) and a COV of 0.32 the characteristic fracture energy can be calculated for different percentage replacements. By comparing the characteristic values computed using Eqs.
(5.1) and (5.2) with the experimental minimum fracture energies for different replacements, it is found that this value encloses the experimental minimum fracture energy (Fig. 5.19).

Fig. 5.19 Variation of characteristic fracture energy and experimental minimum fracture energy with variation in percentage of hematite as coarse aggregate replacing granite in concrete

5.6 Study of the ratio between Mode - II and Mode - I fracture energies

Based on the available test results on Mode - I fracture of concrete, (equation 12 of Ref. [47]). Also, it has been pointed out that the fracture energy in Bazant and Giraudon [47] have proposed, from statistical analysis of test data on Mode - I fracture energy, an equation for the prediction of Mode - I fracture energy ($G_{II}$) of concrete Mode - II can be approximately taken as twenty four times that of Mode - I fracture energy. Though they have not considered hematite as coarse aggregate in concrete, for the sake of comparison the Mode - I fracture energy ($G_{II}$) is computed using equations proposed by them (Appendix B-8) for a given percentage replacement. The ratios between experimental mean $G_{II}$ (i.e., $G_{II}^{e}$) and probabilistic mean $G_{II}$ (i.e., $G_{II}^{p}$) to $G_{II}$ (computed using equation given in [47]) are found out and are presented in
Table 5.6 for different percentages of replacements. The average value of the ratio between $G_{IF}^c$ and $G_{II}^f$ is obtained as 24.72 and between $G_{II}^p$ and $G_{IF}^f$ is 24.68. A good agreement between these two ratios is expected since, as has been noted, there is a good agreement between the results of experimental and probabilistic values of $G_{II}^f$ (Tables 5.3 and 5.4). Also, as already reported in the literature, $G_{II}^f$ values are more than $G_{IF}^f$. The increased fracture energy in Mode-II could be due to extensive aggregate interlock forces that are not present in Mode-I (Swartz et al, [19]). This clearly indicates the chosen DCN specimen geometry fails in predominant Mode–II fracture. From the results it is found that, on the average, the shear fracture energy would be about 24 to 25 times the Mode-I fracture energy, suggesting that the equation proposed by Bazant and Giraudon [47] is valid in the present context also. Also according to Reinhardt and Xu [30] this ratio can be between 20 to 25 for normal strength concrete.

### Table 5.6 – Comparison of mean values of $G_{IF}^f$ with $G_{II}^c$ and $G_{II}^p$

<table>
<thead>
<tr>
<th>Percentage of replacement with hematite aggregate in place of granite aggregate</th>
<th>Compressive strength $f'_c$ (MPa)</th>
<th>Experimental mean $G_{II}^c$ (N/m)</th>
<th>Probabilistic mean $G_{II}^p$ (N/m)</th>
<th>Mean $G_{IF}^f$ (N/m)</th>
<th>$G_{II}^c/G_{IF}^f$</th>
<th>$G_{II}^p/G_{IF}^f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>21.44</td>
<td>1540.30</td>
<td>1540.77</td>
<td>89.33</td>
<td>17.29</td>
<td>17.24</td>
</tr>
<tr>
<td>25</td>
<td>23.16</td>
<td>1755.13</td>
<td>1739.05</td>
<td>92.56</td>
<td>18.96</td>
<td>18.79</td>
</tr>
<tr>
<td>50</td>
<td>23.71</td>
<td>2247.72</td>
<td>2215.71</td>
<td>93.57</td>
<td>24.02</td>
<td>23.68</td>
</tr>
<tr>
<td>75</td>
<td>24.94</td>
<td>2669.87</td>
<td>2719.49</td>
<td>95.77</td>
<td>27.88</td>
<td>28.40</td>
</tr>
<tr>
<td>100</td>
<td>27.21</td>
<td>3534.19</td>
<td>3519.98</td>
<td>99.68</td>
<td>35.46</td>
<td>35.31</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>24.72</td>
<td>24.68</td>
</tr>
</tbody>
</table>
† - The mean value of $G_{if}$ is computed using best fit equation proposed by Bazant and Giraudon[47]