Chapter 5

Reflection and transmission of plane waves at an interface between elastic and micropolar thermoelastic diffusion media

5.1 Introduction

Tomar and Gogna (1992) studied the reflection and refraction of a longitudinal microrotational wave at an interface between two micropolar elastic media in welded contact. Tomar and Gogna (1995 a, b) investigated the problems of reflection and refraction of a longitudinal displacement wave and coupled waves at an interface between two dissimilar micropolar elastic solids. Kumar and Singh (1997) studied the reflection and transmission of elastic waves at a loosely bonded interface between a elastic solid half space and a micropolar elastic solid half space. Singh and Kumar (1998) discussed the problem of reflection and refraction of plane waves at an interface between micropolar elastic solid half space and viscoelastic solid half space. Singh (2001) studied the reflection and refraction of micropolar thermoelastic waves at an interface between a liquid half space and a micropolar generalized thermoelastic solid half space. Kumar and Tomar (2001) discussed the reflection and transmission of elastic waves at viscous liquid/micropolar elastic solid interface.

Singh (2002) discussed reflection and transmission of plane waves at a loosely bonded interface between two dissimilar micropolar viscoelastic solid half spaces. Song,
Zhang, Xu, and Lu (2006) studied the reflection and refraction of micropolar magneto-thermo-viscoelastic waves at the interface between two micropolar viscoelastic media.

Othman and Song (2007) studied the reflection and refraction of thermo-viscoelastic waves at the interface between two micropolar viscoelastic media without energy dissipation. The problem of reflection and transmission of plane waves between two micropolar viscoelastic generalized thermoelastic half spaces of different micropolar, thermal and viscous properties has been investigated by Kumar and Sharma (2008). Kumar, Sharma and Ram (2008b) discussed the reflection and transmission at an imperfect interface between two dissimilar micropolar thermoelastic half spaces. The amplitude ratios for various reflected and transmitted waves have been calculated for an imperfect boundary.

Kumar, Sharma and Ram (2008) studied the propagation of micropolar elastic waves at the imperfect boundary. Kumar, Sharma and Ram (2009b) studied the effect of stiffness on the reflection and transmission of micropolar thermoelastic waves at the interface between an elastic and micropolar generalized thermoelastic solid.

Khurana and Tomar (2008a) discussed the transmission of longitudinal waves at a plane interface between micropolar elastic and Chiral solid half spaces for the waves incident for micropolar half space. Kumar and Panchal (2010) studied the response of loose bonding on reflection and transmission of elastic waves at interface between elastic solid half space and micropolar porous cubic crystal. Kumar, Kaur and Rajvanshi (2011) discussed the reflection and transmission of plane waves at an interface of a micropolar generalized thermoelastic solid half space and a heat conducting micropolar fluid half-space. Kumar and Gupta (2012) investigated the reflection and transmission at a plane interface between an elastic solid half space and a micropolar thermoelastic solid half space with fractional order derivative.

The present chapter deals with the problem of reflection and transmission phenomena due to longitudinal and transverse waves incident obliquely at a plane interface between a uniform elastic solid and micropolar thermoelastic diffusion solid half space. It is found that the amplitude ratios of various reflected and refracted waves are functions of angle of incidence, frequency and are influenced by the micropolarity of the media. The variations of energy ratios with angle of incidence are shown graphically. The sum of all energy ratios of
the reflected waves, transmitted wave and interference between transmitted waves for both incident P- and SV- waves is verified to be unity, which ensures the law of conservation of incident energy at the interface. The conservation of energy at the interface is verified. A special case is also deduced.

5.2 Basic equations

Following Aouadi (2009) and Kumar et al. (2011), the basic equations in homogeneous isotropic micropolar generalized thermoelastic diffusion media in the absence of body forces, body couple, heat and mass diffusion sources are

\[(\lambda + \mu) \nabla (\nabla \cdot \mathbf{u}) + (\mu + K) \nabla^2 \mathbf{u} + K \nabla \times \phi - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla T - \beta_2 \left(1 + \tau_2 \frac{\partial}{\partial t}\right) \nabla C = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \tag{5.2.1}\]

\[(\alpha + \beta + \gamma) \nabla (\nabla \cdot \phi) - \gamma \nabla \times \nabla \times \phi + K \nabla \times \mathbf{u} - 2K \phi = \rho \frac{\partial^2 \phi}{\partial t^2}. \tag{5.2.2}\]

\[\rho \mathcal{C}_E \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + \beta_1 \mathcal{T}_0 \left(\frac{\partial}{\partial t} + \Omega \tau_0 \frac{\partial^2}{\partial t^2}\right) \nabla \cdot \mathbf{u} + a \mathcal{T}_0 \left(\frac{\partial}{\partial t} + \varepsilon \frac{\partial^2}{\partial t^2}\right) C = K^* \nabla^2 T, \tag{5.2.3}\]

\[D \beta_2 \nabla^2 \mathbf{u} + Da \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla^2 T + \left(\frac{\partial}{\partial t} + \Omega t^0 \frac{\partial^2}{\partial t^2}\right) C - Db \left(1 + \tau_2 \frac{\partial}{\partial t}\right) \nabla^2 C = 0, \tag{5.2.4}\]

\[t_{ij} = \lambda \ u_{i,r} \delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr}) - \beta_1 (T + \tau_1 \frac{\partial T}{\partial t}) \delta_{ij} - \beta_2 (C + \tau_2 \frac{\partial C}{\partial t}) \delta_{ij}, \tag{5.2.5}\]

\[m_{ij} = \alpha \phi_{r,j} + \beta \phi_{i,j} + \gamma \phi_{j,i}, \tag{5.2.6}\]

where \(\beta_1 = (3\lambda + 2\mu + K) \alpha \), \(\beta_2 = (3\lambda + 2\mu + K) \alpha\), and \(\alpha\) are, respectively, the coefficients of linear thermal expansion and diffusion expansion, \(\lambda, \mu, \alpha, \beta, \gamma\) and \(\kappa\) are
material constants, \( j \) is the microinertia density, \( K^* \) is the coefficient of thermal conductivity, \( \varphi = (\phi_1, \phi_2, \phi_3) \) is the microrotation vector, \( t_{ij} \) are the components of stress tensor, \( D \) is the thermoelastic diffusion constant, \( \nabla \) and \( \nabla^2 \) are, respectively, the gradient and Laplacian operators, \( \delta_{ij} \) is Kronecker delta, \( \tau_0, \tau_1 \) are thermal relaxation times with \( \tau_i \geq \tau_0 \geq 0 \) and \( \tau_0, \tau^1 \) are diffusion relaxation times with \( \tau^1 \geq \tau^0 \geq 0 \). Here \( \tau_1 = \tau^1 = 0, \Omega = 1, \varepsilon = \tau_0 \) for Lord-Shulman (L-S) model and \( \Omega = 0, \varepsilon = \tau^0 \) for Green-Lindsay model and the other symbols used in the equation (5.2.1)-(5.2.6) are as given in section 2.2 of Chapter 2.

Following Bullen (1963), the equations of motion in homogeneous isotropic elastic solid medium are

\[
(\lambda^e + \mu^e)\nabla(\nabla \cdot \mathbf{u}^e) + \mu \nabla^2 \mathbf{u}^e = \rho^e \frac{\partial^2 \mathbf{u}^e}{\partial t^2},
\]

(5.2.7)

where \( \lambda^e, \mu^e \) are Lame’s constants, \( \mathbf{u}^e = (u^e, v^e, w^e) \), \( \rho^e \) are, respectively, the displacement vector and density corresponding to the isotropic elastic solid.

The stress-strain relation in the isotropic elastic medium is given by

\[
\sigma_{ij}^e = \lambda^e \epsilon_{k,k}^e \delta_{ij} + 2\mu^e \epsilon_{ij}^e,
\]

(5.2.8)

where \( \sigma_{ij}^e \) and \( \epsilon_{ij}^e = \frac{1}{2} (u^e_{i,j} + u^e_{j,i}) \) are, respectively components of stress and strain tensor, \( \epsilon_{k,k}^e \) is the dilatation.

### 5.3 Formulation of the Problem

We consider an isotropic elastic solid half space (Medium I) lying over a homogenous isotropic, micropolar generalized thermoelastic diffusion solid half space (Medium II). The origin of the Cartesian coordinate system \((x, y, z)\) is taken at any point on the plane surface \( z = 0 \) and \( z \)–axis points vertically downwards into the micropolar thermoelastic diffusion solid half space. The elastic solid half space occupies the region \( z \leq 0 \) (Medium I) and the
region $z \geq 0$ is occupied by the micropolar thermoelastic diffusion solid half space (Medium II) as shown in Figure 5.1. For two-dimensional problem, we assume the displacement vector $u^e$ in Medium I and the displacement vector $u$ and microrotation vector $\varphi$ in Medium II as

$$ u^e = (u^e, 0, w^e), \quad u = (u, 0, w), \quad \varphi = (0, \varphi_y, 0). $$

Equations (5.2.1)-(5.2.4) with the aid of equation (5.3.1) take the form

$$(\mu + K)u_{,xx} + (\lambda + \mu)u_{,xx} + (\mu + K)u_{,zz} + (\lambda + \mu)w_{,xz} + K\phi_{y,z} - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T_{,x} - \beta_2 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) C_{,x} = \rho \ddot{u},$$

$$(\mu + K)w_{,xx} + (\lambda + \mu)u_{,xz} + (\mu + K)w_{,zz} + (\lambda + \mu)w_{,xz} - K\phi_{y,x} - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T_{,z} - \beta_2 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) C_{,z} = \rho \ddot{w},$$

$$\gamma(\phi_{y,x} + \phi_{y,z}) - 2K\phi_y + K(w_{,1} - u_{,3}) = \rho j \dot{\phi}_y$$

$$\rho C_E \left[1 + \tau_0 \frac{\partial}{\partial t}\right] \dot{T} + \beta_2 T_0 \left(1 + \Omega_0 \frac{\partial}{\partial t}\right) \left(u_{,x} + \dot{w}_{,z}\right) + a T_0 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \dot{C} =$$

$$K^* (T_{,xx} + T_{,zz}),$$

$$D\beta_2 (u_{,xxx} + w_{,xxx} + u_{,zzz} + w_{,zzz}) + D\alpha \left[1 + \tau_1 \frac{\partial}{\partial t}\right] (T_{,xx} + T_{,zz}) +$$

$$\left(1 + \Omega_0 \tau_0 \frac{\partial}{\partial t}\right) \dot{C} - D\beta \left(1 + \tau_0 \frac{\partial}{\partial t}\right) (C_{,xx} + C_{,zz}) = 0.$$

We define dimensionless quantities as

$$x' = \frac{\alpha}{\alpha^*}, \quad y' = \frac{\omega}{\omega^*}, \quad u' = \frac{u}{u^e}, \quad w' = \frac{w}{w^e}, \quad \tau' = \frac{\tau}{\tau^0}, \quad t' = \frac{t}{t^0},$$

$$z' = \frac{z}{z^0}, \quad \rho' = \frac{\rho}{\rho^*}, \quad k' = \frac{k}{k^*}, \quad T' = \frac{T}{T^*}, \quad C' = \frac{C}{C^*}.$$
\[ \phi'_y = \frac{\rho v^2_1}{\beta_1 T_0} \phi_y, \quad T' = \frac{T}{T_0}, \quad C' = \frac{\beta_2 C}{\beta_1 T_0} \]

\[ m'_{zy} = \frac{\omega^*}{v_1 \beta_1 T_0} m_{zy}, \quad P_{ij}' = \frac{\rho v^2_1}{\beta_1^2 T_0} P_{ij}^*, P^{*e} = \frac{\rho v^2_1}{\beta_1^2 T_0} P^{*e}, \quad (5.3.7) \]

where

\[ \omega^* = \frac{\rho C E v^2_1}{K^2}, \quad v^2_1 = \frac{\lambda + 2\mu + K}{\rho}. \]

Using the dimensionless quantities defined by equation (5.3.7) in equations (5.3.2)-(5.3.6), we obtain

\[ \delta_1(u_{,xx} + u_{,zz}) + \delta_2(u_{,xx} + w_{,zz}) + \delta_3 \phi_{,zz} - \tau^1_{,T,xx} - \tau^1_{,C,xx} = \ddot{u}, \quad (5.3.8) \]

\[ \delta_1(w_{,xx} + w_{,zz}) + \delta_2(u_{,zz} + w_{,zz}) - \delta_3 \phi_{,x} - \tau^1_{,T,zz} - \tau^1_{,C,zz} = \ddot{w}, \quad (5.3.9) \]

\[ \nabla^2 \phi_2 + a_1 (u_{,zz} - w_{,zz}) - 2a_1 \phi_y = a_2 \ddot{\phi}_y, \quad (5.3.10) \]

\[ \zeta_1 \tau^0_{,T} (\dot{u}_{,x} + \dot{w}_{,z}) + \tau^0_{,T} \tau^0_{,T} + \zeta_2 \tau^0_{,T} \dot{C} = \nabla^2 T, \quad (5.3.11) \]

\[ q'^*_1 (u_{,xxx} + u_{,xxx} + u_{,xxx} + w_{,xxx}) + q'^*_2 \tau^1_{,T,xx} + T_{,xx} - q'^*_3 \tau^1_{,C,xx} + \tau^0_{,C} \dot{C} = 0, \quad (5.3.12) \]

where

\[ (\delta_1, \delta_2, \delta_3) = \frac{1}{\rho v^2_1} \left( \mu + K, \lambda + \mu, K \right), \quad (a_1, a_2) = \frac{v^2_1}{\gamma} \left( \frac{K}{\omega^*}, \rho \right), \]

\[ (\tau^1_1, \tau^1_{,T}, \tau^1_{,T}, \tau^0_{,T}, \tau^0_{,T}) = \left( 1 + \tau_1 \frac{\partial}{\partial t}, 1 + \tau_1 \frac{\partial}{\partial t}, 1 + \tau_0 \frac{\partial}{\partial t}, 1 + \tau_0 \frac{\partial}{\partial t}, 1 + \Omega \tau_0 \frac{\partial}{\partial t}, 1 + \Omega \tau_0 \frac{\partial}{\partial t} \right), \]

\[ (q'^*_1, q'^*_2, q'^*_3) = \frac{D \omega^*}{v^2_1} \left( \frac{\beta_2^2}{\rho v^2_1}, \frac{\beta_2 a}{\beta_1}, b \right) \left( \zeta_1, \zeta_2 \right) = \frac{\beta_1 T_0}{K^{*e} \omega^*} \left( \frac{\beta_1}{\rho \beta_2} \right) \]

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The displacement components \( u \) and \( w \) are related by the potential functions \( \phi \) and \( \psi \) through the relations

\[
\begin{align*}
  u &= \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \\
  w &= \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}.
\end{align*}
\]  

(5.3.13)

Substituting the value of \( u \) and \( w \) from equation (5.3.13) in equations (5.3.8)-(5.3.12), we obtain

\[
\begin{align*}
  \left( \nabla^2 - \frac{\partial^2}{\partial t^2} \right) \phi - \tau_1^1 T - \tau_1^1 C &= 0, \\
  \left( \nabla^2 - a_3 \frac{\partial^2}{\partial t^2} \right) \psi + a_4 \phi_y &= 0, \\
  \left( \nabla^2 - 2a_1 - a_2 \frac{\partial^2}{\partial t^2} \right) \phi_y - a_1 \nabla^2 \psi &= 0, \\
  \left( \nabla^2 - \tau_1^0 \frac{\partial}{\partial t} \right) T - \zeta_2 \tau_1^0 \dot{C} - \zeta_1 \tau_1^0 \nabla^2 \phi &= 0, \\
  q_i^* \nabla^4 \phi + q_i^* \tau_i^1 \nabla^2 T - q_i^* \tau_i^1 \nabla^2 C + \tau^0_{cc} \dot{C} &= 0,
\end{align*}
\]

(5.3.14) \hspace{1cm} (5.3.15) \hspace{1cm} (5.3.16) \hspace{1cm} (5.3.17) \hspace{1cm} (5.3.18)

where

\[
(a_3, a_4) = \frac{1}{\mu + K} \left( \rho \nu_1^2, K \right),
\]

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.
\]
Assuming the motion to be time harmonic, we take
\[
\phi_0, \psi_0, \phi_y, T, C \right\} (x, z, t) = \{ \phi, \psi, \phi \phi_y, \bar{T}, \bar{C} \} \exp^{-i\omega t}, \tag{5.3.19}
\]
where \( \omega \) is the angular frequency of the vibrations of material particles.

Substituting the values of \( \phi, \psi, \phi_y, T \) and \( C \) from equation (5.3.19) in equations (5.3.14)- (5.3.18), we obtain
\[
(\nabla^2 + \omega^2)\bar{\phi} - \tau_1^{11} \bar{T} - \tau_c^{11} \bar{C} = 0, \tag{5.3.20}
\]
\[
(\nabla^2 + a_3 \omega^2)\bar{\psi} + a_4 \phi_y = 0, \tag{5.3.21}
\]
\[
(\nabla^2 - 2a_1 + a_2 \omega^2)\phi_y - a_1 \nabla^2 \bar{\psi} = 0, \tag{5.3.22}
\]
\[
-\zeta_1 \tau_{tt}^0 \nabla^2 \phi - (\nabla^2 - \tau_{t}^{10})T - \zeta_2 \tau_{c}^{10} \bar{C} = 0, \tag{5.3.23}
\]
\[
q_1^* \nabla^4 \bar{\phi} + q_2^* \tau_1^{11} \nabla^2 \bar{T} - (q_3^* \tau_c^{11} \nabla^2 - \tau_{c c}^{10})\bar{C} = 0, \tag{5.3.24}
\]

where
\[
(\tau_1^{11}, \tau_c^{11}, \tau_t^{10}, \tau_{c c}^{10}, \tau_a^{10}) = \left( 1 - i\omega \tau_1, 1 - i\omega \tau_1^1, 1 - i\omega \tau_2, -i\omega \tau_0, -i\omega (1 - i\omega \tau_1), -i\omega (1 - i\omega \tau_2) \right).
\]

Equation (5.3.23) and (5.3.24) of this system are solved into two relations, given by
\[
[q_3^* \tau_c^{11} \tau_1^{10} + q_1^* \tau_c^{10} \tau_2^0] \nabla^4 \phi = -[q_1^* \tau_{tt}^0 \tau_{cc}^{10} \nabla^2 - \zeta_1 \tau_{tt}^0 \tau_{cc}^{10} \nabla^2] \phi = 0, \tag{5.3.25}
\]
\[
[q_3^* \tau_c^{11} \nabla^4 - (q_3^* \tau_c^{11} \tau_1^{10} + \tau_{cc}^{10} + q_2^* \tau_c^{11} \tau_2^0) \nabla^2 + \tau_c^{10} \tau_{cc}^{10}] \nabla T, \tag{5.3.25}
\]
\[
[q_1^* \nabla^6 + [(q_2^* \tau_t^{11} \tau_1^{10} - q_1^* \tau_t^{10}) \nabla^4] \bar{\phi} = 0, \tag{5.3.26}
\]
\[
[q_3^* \tau_c^{11} \nabla^4 - (q_3^* \tau_c^{11} \tau_1^{10} + \tau_{cc}^{10} + q_2^* \tau_c^{11} \tau_2^0) \nabla^2 + \tau_c^{10} \tau_{cc}^{10}] \nabla \bar{C}. \tag{5.3.26}
\]

Making use of equations (5.3.25) and (5.3.26) in equation (5.3.20), we obtain
\[
[A_1 \nabla^6 + A_2 \nabla^4 + A_3 \nabla^2 + A_4] \bar{\phi} = 0, \tag{5.3.27}
\]

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where

\[ A_1 = (q_1^* - q_3^*)r_{cc}^{11}, \quad A_2 = \tau_{cc}^{10} + (q_1^* + q_2^*)\tau_{c}^{10} r_{t}^{11} \zeta_2 + (q_2^* + q_3^*)\tau_{tt}^{10} r_{t}^{11} \zeta_1. \]

\[ A_3 = (\omega^2 + \tau_{c}^{10} \zeta_1^2 - \zeta_1^2 \tau_{tt}^{10} r_{t}^{11}) r_{cc}^{10} + q_2^* \zeta_2^{11} r_{c}^{10} + \omega^2 q_3^* r_{c}^{11} \tau_{t}^{10}, \quad A_4 = -\omega^2 r_{cc}^{10} \tau_{tt}^{10}. \]

Substituting the value of \( \bar{\phi}_y \) from equation (5.3.22) in equation (5.3.21), we obtain

\[ [\nabla^4 + H_1 \nabla^2 + H_2 \bar{\psi} = 0, \quad (5.3.28) \]

where

\[ H_1 = a_1a_4 - 2a_1 + (a_2 + a_3)\omega^2, \quad H_2 = (a_2\omega^2 - 2a_1)\omega^2. \]

The general solution of equation (5.3.27) can be written as

\[ \bar{\phi} = \sum_{i=1}^{3} \bar{\phi}_i, \quad (5.3.29) \]

where the potentials \( \bar{\phi}_i \), \( i = 1,2,3 \) are solutions of wave equations, given by

\[ \left[ \nabla^2 + \frac{\omega^2}{V_i^2} \right] \bar{\phi}_i = 0, \quad i = 1,2,3. \quad (5.3.30) \]

Here \( V_i^2 \) (\( i = 1,2,3 \)) are the velocities of three longitudinal waves, that is, Longitudinal displacement (LD) wave, Thermal (T) wave and Mass diffusion (MD) wave and derived from the roots of cubic equation in \( V^2 \) given by

\[ A_4V^6 - A_3\omega^2V^4 + A_2\omega^4V^2 - A_1\omega^6 = 0. \quad (5.3.31) \]

The general solution of equation (5.3.28) can be written as

\[ \bar{\psi} = \sum_{i=4}^{5} \bar{\psi}_i, \quad (5.3.32) \]

where the potentials \( \bar{\psi}_i \), \( i = 4,5 \) are solutions of wave equations, given by
\[
\left[ \nabla^2 + \frac{\omega^2}{V_i^2} \right] \bar{\psi}_i = 0, \quad i = 4,5. \tag{5.3.33}
\]

Here \( V_i^2 \) \((i = 4,5)\) are the velocities of two transverse displacement waves coupled with transverse microrotation (CD I, CD II) and are derived from the roots of the equation in \( V^2 \) given by

\[
H_2 V^4 - H_1 \omega^2 V^2 + \omega^4 = 0. \tag{5.3.34}
\]

Making use of equation (5.3.29) in the equations (5.3.25), (5.3.26), with the aid of equations (5.3.19) and (5.3.30), the general solutions for \( \phi, T \) and \( C \) are obtained as

\[
\{ \phi, T, C \} = \sum_{i=1}^{3} \{1, r_{ii}, r_{2i}\} \phi_i e^{-i\omega t}, \tag{5.3.35}
\]

where

\[
r_{ii} = \left[ \left\{ q^*_3 \tau^* c \zeta^*_1 \tau^*_{ii} + q^*_1 \tau^* c \zeta^*_2 \right\} \omega^4 + \zeta^*_1 \tau^*_{ii} \tau^*_{cc} \omega^2 V_i^2 \right] / p_{ii},
\]

\[
r_{2i} = \left[ -q^*_1 \omega^6 + \left\{ q^*_2 \tau^*_{ii} + q^*_3 \tau^* c \zeta^*_2 \right\} \omega^4 V_i^2 \right] / V_i^2 p_{ii},
\]

and

\[
p_{ii} = \left[ q^*_3 \tau^* c \omega^4 + \left\{ q^*_1 \tau^*_{ii} + q^*_2 \tau^*_{cc} \right\} \omega^2 V_i^2 + \tau^*_{ii} \tau^*_{cc} V_i^4 \right], \quad i = 1,2,3.
\]

Substituting equation (5.3.32) in the equations (5.3.21)-(5.3.22), with the aid of equations (5.3.19) and (5.3.33), the general solutions for \( \psi \) and \( \phi_2 \) are obtained as

\[
\{ \psi, \phi_2 \} = \sum_{i=4}^{5} \{1, p_{ii}\} \psi_i e^{-i\omega t}, \tag{5.3.36}
\]

where

\[
p_{ii} = \frac{a_1 \omega^2}{(a_2 \omega^2 - 2a_1)V_i^2 - \omega^2}, \quad i = 4,5. \tag{5.3.37}
\]

Using the dimensionless quantities defined by (5.3.7) in the equation (5.2.7) and with the aid of the equation (5.3.1), after suppressing the primes, we obtain
where \( \eta^e = \sqrt{\frac{\chi^e + 2\mu^e}{\rho^e}} \), \( \chi^e = \sqrt{\frac{\mu^e}{\rho^e}} \) are, respectively, the velocities of longitudinal (P) wave and transverse waves (SV) wave of the medium I.

The components of displacement \( u^e \) and \( w^e \) are related by the potential functions as

\[
u^e = \frac{\partial \phi^e}{\partial x} - \frac{\partial \psi^e}{\partial z}, \quad w^e = \frac{\partial \phi^e}{\partial z} + \frac{\partial \psi^e}{\partial x},
\]

(5.3.40)

where \( \phi^e \) and \( \psi^e \) are solutions of wave equations

\[\nabla^2 \phi^e = \frac{\ddot{\phi}^e}{\bar{\eta}^2}, \quad \nabla^2 \psi^e = \frac{\ddot{\psi}^e}{\bar{\chi}^2},\]

(5.3.41)

where

\[\bar{\eta} = \frac{\eta^e}{v_1}, \quad \bar{\chi} = \frac{\chi^e}{v_1} .\]

### 5.4 Reflection and Transmission

We consider a plane wave (P or SV) propagating through the isotropic elastic solid half space and is incident at the interface \( z = 0 \), as shown in Figure 5.1. Corresponding to each incident wave, two waves (P and SV) are reflected in isotropic elastic solid half space and five waves (LD, T, MD, CD I and CD II) are transmitted in isotropic micropolar thermoelastic diffusion solid half space.
Incident (P or SV)

Elastic half space
(Medium I) $z<0$

Contact surface $z=0$

Micropolar thermelastic diffusion half space
(Medium II) $z>0$

Figure 5.1 Geometry of the problem
In elastic solid half space (Medium I), a set of plane wave solutions for the displacement potential functions satisfying equation (5.3.41) can be written as

\[ \phi^e = A_0^e e^{i\omega \left[ (x \sin \theta_1 + z \cos \theta_1) / \hat{n} - t \right]} + A_1^e e^{i\omega \left[ (x \sin \theta_1 + z \cos \theta_1) / \tilde{n} - t \right]}, \]  
(5.4.1)

\[ \psi^e = B_0^e e^{i\omega \left[ (x \sin \theta_1 + z \cos \theta_1) / \tilde{x} - t \right]} + B_1^e e^{i\omega \left[ (x \sin \theta_1 + z \cos \theta_1) / \tilde{x} - t \right]}, \]  
(5.4.2)

The coefficients \( A_0^e (B_0^e), A_1^e \) and \( B_1^e \) represent the amplitude of the incident P (or SV), reflected P and reflected SV waves respectively.

Following Borcherdt (1982), in isotropic micropolar thermoelastic diffusion solid half space, the potential functions satisfying equations (5.3.30) and (5.3.33) can be written as

\[ \{ \phi, T, C \} = \sum_{i=1}^{3} \{ 1, \eta_i, r_{2i} \} B_i e^{i(\hat{A}_i \cdot \hat{r})} e^{i(\tilde{p}_i \cdot \hat{r} - \omega \tau)}, \]  
(5.4.3)

\[ \{ \psi, \phi_y \} = \sum_{i=4}^{5} \{ 1, \rho_{ii} \} B_i e^{i(\hat{A}_i \cdot \hat{r})} e^{i(\tilde{p}_i \cdot \hat{r} - \omega \tau)}. \]  
(5.4.4)

The coefficients \( B_i, (i = 1,2,3,4,5) \) represent the amplitudes of transmitted waves. The propagation vector \( \tilde{p}_i, i = 1,2,3,4,5 \) and attenuation factor \( \tilde{A}_i, i = 1,2,3,4,5 \) are given by

\[ \tilde{p}_i = \xi_R \hat{x} + dV_{iR} \hat{z}, \tilde{A}_i = -\xi_I \hat{x} - dV_{iI} \hat{z}, i = 1,2,3,4,5 \]  
(5.4.5)

where

\[ dV_i = dV_{iR} + idV_{iI} = p.v. \left( \frac{\omega^2}{V_i^2} - \xi^2 \right), i = 1,2,3,4,5. \]  
(5.4.6)

and \( \xi = \xi_R + i\xi_I \) is the complex wave number. The subscripts \( R \) and \( I \) denote the real and imaginary parts of the corresponding complex number and p.v. stands for the principal value of the complex quantity obtained from square root. \( \xi_R \geq 0 \) ensures propagation in positive \( x \)-direction. The complex wave number \( \xi \) in the isotropic micropolar thermoelastic diffusion medium is given by

\[ \xi = \left| \tilde{p}_i \right| \sin \tilde{\theta}_i - i\left| \tilde{A}_i \right| \sin(\tilde{\theta}_i - \tilde{\gamma}_i), \quad i = 1,2,3,4,5 \]  
(5.4.7)

where \( \tilde{\gamma}_i (i = 1,2,3,4,5) \) is the angle between the propagation and attenuation vector and \( \tilde{\theta}_i (i = 1,2,3,4,5) \) is the angle of transmission in medium II.
5.5 Boundary conditions

The boundary conditions to be satisfied at the interface $x_3 = 0$ as follows:

(I) Continuity of stress components

$$t^e_{zz} = t_{zz},$$

$$t^e_{zx} = t_{zx},$$

(5.5.1)

(II) Continuity of displacement components

$$w^e = w,$$

$$u^e = u,$$

(5.5.3)

(5.5.4)

(III) Vanishing of the tangential couple stress component

$$m_{zy} = 0,$$

(5.5.5)

(IV) Thermally insulated boundary

$$\frac{\partial T}{\partial z} = 0,$$

(5.5.6)

(V) Impermeable boundary

$$\frac{\partial C}{\partial z} = 0.$$  

(5.5.7)

Making use of equations (5.4.1)-(5.4.4) in equations (5.5.1)-(5.5.7), we find that the boundary conditions are satisfied if and only if

$$\xi_R = \frac{\omega \sin \theta_0}{V_0} = \frac{\omega \sin \theta_1}{\bar{n}} = \frac{\omega \sin \theta_2}{\bar{X}},$$

(5.5.8)

and

$$\xi_I = 0,$$

(5.5.9)

where

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\[ V_0 = \begin{cases} \tilde{\eta}, & \text{for incident } P-\text{wave} \\ \tilde{\chi}, & \text{for incident } SV-\text{wave} \end{cases} \]  

(5.5.10)

It means that waves are attenuating only in \( z \)-direction. From equation (5.4.7), it implies that if \( \tilde{\mathbf{A}} \neq 0 \) then \( \tilde{\gamma}_i = \tilde{\theta}_i \) (\( i = 1,2,3,4,5 \)) that is, attenuated vectors for the five transmitted waves are directed along the \( z \)-axis.

Substituting the values of \( \phi^e, \psi^e, \phi, T, C, \psi \) and \( \phi_y \) from equations (5.4.1)-(5.4.4) in equations (5.5.1)-(5.5.7) and with the aid of the equations (5.2.5),(5.2.6),(5.2.8), (5.3.1),(5.3.7),(5.3.13),(5.3.40),(5.5.8) and (5.5.9), we obtain a system of seventh non-homogeneous equations which can be written as

\[ \sum_{j=1}^{7} f_{ij}Z_j = d_j, \]  

(5.5.11)

where \( Z_j = |Z_j|^i\psi_j^*|Z_j|^\psi_j \), \( j = 1,2,3,4,5,6,7 \) are respectively, the ratios of amplitudes and phase shift of reflected \( P^- \), reflected \( SV^- \), transmitted \( LD,T,MD,CDI \) and \( CDII \) waves to that of incident wave respectively.

\[
\begin{align*}
  f_{11} &= 2\mu^e\left(\frac{\varepsilon_R}{\omega}\right)^2 - \rho^e \nu_1^2, \\
  f_{12} &= 2\mu^e\left(\frac{\varepsilon_R \omega}{\omega^2}\frac{dV}{\omega}\right) - 2\mu^e\left(\frac{\varepsilon_R}{\omega}\right)^2, \\
  f_{16} &= (2\mu + K)\left(\frac{\varepsilon_R}{\omega}\right)^2 - \frac{dV}{\omega^2}, \\
  f_{17} &= (2\mu + K)\left(\frac{\varepsilon_R \omega}{\omega^2}\frac{dV}{\omega}\right), \\
  f_{21} &= 2\mu^e\left(\frac{\varepsilon_R}{\omega}\right)^2, \\
  f_{22} &= \mu^e\left(\frac{dV}{\omega}\right)^2 - \left(\frac{\varepsilon_R}{\omega}\right)^2, \\
  f_{26} &= \mu\left(\frac{\varepsilon_R}{\omega}\right)^2, \\
  f_{27} &= \mu\left(\frac{\varepsilon_R}{\omega}\right)^2, \\
  f_{32} &= \frac{dV}{\omega^2}, \\
  f_{33} &= \frac{dV}{\omega}, \\
  f_{34} &= -\frac{dV}{\omega}, \\
  f_{42} &= -\frac{dV}{\omega}, \\
  f_{44} &= -\frac{dV}{\omega}, \\
  f_{51} &= 0, \\
  f_{52} &= 0, \\
  f_{55} &= 0, \\
  f_{61} &= 0, \\
  f_{62} &= 0, \\
  f_{66} &= 0, \\
  f_{71} &= 0, \\
  f_{72} &= 0, \\
  f_{76} &= 0, \\
  f_{77} &= 0, \\
  f_{1j} &= \lambda\left(\frac{\varepsilon_R}{\omega}\right)^2 + \nu_1^2\left(\frac{dV}{\omega}\right)^2, \\
  f_{2j} &= 2\mu\left(\frac{\varepsilon_R}{\omega}\right)^2, \\
  f_{3j} &= -\frac{\varepsilon_R}{\omega^2}, \\
  f_{4j} &= -\frac{dV}{\omega}, \\
  f_{5j} &= r_j \frac{dV}{\omega}, \\
  f_{6j} &= r_j \frac{dV}{\omega}, \\
  f_{7j} &= p_j \frac{dV}{\omega}, \\
  j &= 3,4,5
\end{align*}
\]
\[
\frac{dV_{\eta}}{\omega} = \sqrt{\frac{1}{\eta^2} - \left(\frac{\xi}{\omega}\right)^2} = \sqrt{\frac{1}{\eta^2} - \frac{\sin^2 \theta_0}{V_0^2}}, \\
\frac{dV_{\chi}}{\omega} = \sqrt{\frac{1}{\chi^2} - \frac{\sin^2 \theta_0}{V_0^2}},
\]
and
\[
\frac{dV_j}{\omega} = \text{p.v.} \left( \sqrt{\frac{1}{V_j^2} - \frac{\sin^2 \theta_0}{V_0^2}} \right), \quad j = 1,2,3,4,5.
\]

Here \(\text{p.v.}\) is evaluated with restriction \(dV_j \geq 0\) to satisfy decay condition in the micropolar thermoelastic diffusion medium. The coefficients \(d_i,\ i = 1,2,3,4,5,6,7\) on the right side of the equation (5.5.11) are given by

(I) For incident P-wave
\[
d_i = (-1)^i f_{i1} \text{ for } i = 1,2,3,4 \text{ and } d_i = 0 \text{ for } i = 5,6,7. \tag{5.5.12}
\]

(II) For incident SV-wave
\[
d_i = (-1)^{i+1} f_{i2} \text{ for } i = 1,2,3,4 \text{ and } d_i = 0 \text{ for } i = 5,6,7. \tag{5.5.13}
\]

Now we consider a surface element of unit area at the interface between two media. The reason for this consideration is to calculate the partition of energy of the incident wave among the reflected and transmitted waves on the both sides of surface. Following Achenbach (1973), the energy flux across the surface element, that is, rate at which the energy is communicated per unit area of the surface is represented as

\[
P^* = t_{qr} q_r \dot{u}_q, \tag{5.5.14}
\]

where \(t_{qr}\) are the components of stress tensor, \(q_r\) are the direction cosines of the unit normal \(\hat{q}\) outward to the surface element and \(\dot{u}_q\) are the components of the particle velocity.

The time average of \(P^*\) over a period, denoted by \(<P^*>\), represents the average energy transmission per unit surface area per unit time. Thus, on the surface with normal along \(z\)-direction, the average energy intensities of the waves in the elastic solid are given by

\[
<P^{*e}> = \text{Re} <t>_{zz}^{\epsilon} \text{Re}(\dot{u}^{\epsilon}) + \text{Re} <t>_{zz}^{\epsilon} \text{Re}(\dot{w}^{\epsilon}). \tag{5.5.15}
\]
Following Achenbach (1973), for any two complex functions $f$ and $g$, we have

$$< \text{Re}(f) \cdot \text{Re}(g) > = \frac{1}{2} \text{Re}(f \cdot \overline{g}).$$ \hspace{1cm} (5.5.16)

The expressions for energy ratios $E_i$ ($i = 1, 2$) for the reflected P and reflected SV are given by

$$E_i = -\frac{< P_i^{se} >}{< P_0^{se} >}, \quad i = 1, 2$$ \hspace{1cm} (5.5.17)

where

$$< P_1^{se} > = \frac{\omega^4 \rho^e v_1^2}{\eta'} |Z_1|^2 \text{Re} \left( \cos \vartheta_1 \right), \quad < P_2^{se} > = \frac{\omega^4 \rho^e v_1^2}{\chi'} |Z_2|^2 \text{Re} \left( \cos \vartheta_2 \right),$$

and

(I) For incident P- wave

$$< P_0^{se} > = -\frac{\omega^4 \rho^e v_1^2}{\eta'} \cos \vartheta_0.$$ \hspace{1cm} (5.5.18)

(II) For incident SV- wave

$$< P_0^{se} > = -\frac{\omega^4 \rho^e v_1^2}{\chi'} \cos \vartheta_0.$$ \hspace{1cm} (5.5.19)

are the average energy intensities of the reflected P-, reflected SV-, incident P- and incident SV-waves respectively. In equation (5.5.17), negative sign is taken because the direction of reflected waves is opposite to that of incident wave.

For micropolar thermoelastic diffusion solid half space, the average energy intensities of the waves on the surface with normal along $z -$ direction, are given by
\[ < P_{ij}^* > = \Re < t \chi_{ij} > \Re(u^{(j)}(t)) + \Re < t \chi_{ij} > \Re(u^{(i)}(t)) + \Re < m \chi_{iz} > \Re(\dot{\phi}_{yz}^{(i)}). \] (5.5.20)

The expressions for the energy ratios \( E_{ij} \) (\( i,j = 1,2,3,4,5 \)) for the transmitted waves are given by

\[ E_{ij} = \frac{< P_{ij}^* >}{< P_{0e}^* >}, \quad i,j = 1,2,3,4,5 \] (5.5.21)

where

\[ < P_{ij}^* > = -\omega^4 \Re[(2\mu + K) \frac{dV_i}{\omega} \overline{\xi_R} \bar{\overline{\xi_R}} + \{\lambda \left(\frac{\xi_R}{\omega}\right)^2 + \rho v_i^2 \left(\frac{dV_i}{\omega}\right)^2 + \rho v_i^2 (r_i \tau_{i1} + r_i \tau_{i1}) \frac{dV_i}{\omega} \} Z_{ij+2} \bar{Z}_{j+2} ], \]

\[ < P_{4j}^* > = -\omega^4 \Re\left\{ (\mu + K) \left(\frac{dV_4}{\omega}\right)^2 - \mu \left(\frac{\xi_R}{\omega}\right)^2 - \frac{K p_{14}}{\omega^2} \right\} \frac{dV_j}{\omega} + \frac{(2\mu + K) \xi_R dV_4 \overline{\xi_R}}{\omega^2} + \frac{\gamma \omega^2}{\rho v_4^2} \frac{dV_4}{\omega} p_{14} \bar{p}_{1j} \left( Z_6 \bar{Z}_{j+2} \right), \]

\[ < P_{5j}^* > = -\omega^4 \Re\left\{ (\mu + K) \left(\frac{dV_5}{\omega}\right)^2 - \mu \left(\frac{\xi_R}{\omega}\right)^2 - \frac{K p_{15}}{\omega^2} \right\} \frac{dV_j}{\omega} + \frac{(2\mu + K) \xi_R dV_5 \overline{\xi_R}}{\omega^2} + \frac{\gamma \omega^2}{\rho v_5^2} \frac{dV_5}{\omega} p_{15} \bar{p}_{1j} \left( Z_6 \bar{Z}_{j+2} \right), \]

for \( j = 4,5 \).

The diagonal entries of energy matrix \( E_{ij} \) in equation (5.5.21) represent the energy ratios of the LD, MD, T, CD I and CD II waves, whereas sum of the non-diagonal entries of \( E_{ij} \) gives the share of interaction energy among all the transmitted waves in the medium and is given by

\[ E_{RR} = \sum_{i=1}^{5} \left( \sum_{j=1}^{5} E_{ij} - E_{ii} \right) \] (5.5.22)
The energy ratios $E_i$, $i=1,2$, diagonal entries and sum of non-diagonal entries of energy matrix $E_{ij}$, that is, $E_{11}, E_{22}, E_{33}, E_{44}, E_{55}$ and $E_{RR}$ yield the conversation of the incident energy across the interface, through the relation

$$E_1 + E_2 + E_{11} + E_{22} + E_{33} + E_{44} + E_{55} + E_{RR} = 1.$$  

### 5.6 Special case

When the micropolarity effect is neglected from the Medium II, i.e. if we take $K = j = 0$ in equations (5.5.11) and (5.5.21), we obtain corresponding expressions for amplitude and energy ratios of reflected P-wave, reflected SV-wave, transmitted P-wave, transmitted T-wave, transmitted MD-wave, and transmitted SV waves to that of incident wave. In these expressions the velocities $V_1, V_2, V_3$ are derived from the equation (5.3.27) in the absence of micropolarity effect and $V_4 = \sqrt{\delta_1}$ is the velocity of the transverse wave (SV) and coupling constants are given by equation (5.3.35).

### 5.7 Numerical results and Discussion

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results. Following Sherief and Saleh (2005), we take the values of relevant parameters for copper material as

$$\lambda = 7.76 \times 10^{10} \text{ Kgm}^{-1} \text{s}^{-2}, \quad \mu = 3.86 \times 10^{10} \text{ Kgm}^{-1} \text{s}^{-2}, \quad T_0 = 0.293 \times 10^3 \text{ K},$$

$$C_E = 0.3831 \times 10^3 \text{ JKg}^{-1} \text{ K}^{-1}, \quad \alpha_t = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad \alpha_C = 1.98 \times 10^{-4} \text{ Kgm}^{-1} \text{ m}^3,$$

$$a = 1.2 \times 10^4 \text{ m}^2 \text{s}^{-2} \text{ K}^{-1}, \quad b = 9 \times 10^5 \text{ m}^5 \text{s}^{-2} \text{ Kg}^{-1}, \quad D = 0.85 \times 10^{-8} \text{ m}^{-3} \text{sKg},$$

$$\rho = 8.954 \times 10^3 \text{ Kgm}^{-3},$$

The values of micropolar parameters are taken as
\[ K^* = 0.386 \times 10^3 \text{Wm}^{-1}\text{K}^{-1}, \quad K = 1.0 \times 10^{10} \text{N/m}^2, \quad j = 0.2 \times 10^{-19} \text{m}^2, \quad \gamma = 0.779 \times 10^{-9} \text{N}, \]

and the values of relaxation times are
\[ \tau_0 = 0.4 \text{s}, \quad \tau_1 = 0.9 \text{s}, \quad \tau^0 = 0.5 \text{s}, \quad \tau^1 = 0.8 \text{s}. \]

Following Bullen (1963), the numerical data of granite for elastic medium is given by
\[ \rho^e = 2.65 \times 10^3 \text{Kgm}^{-3}, \quad \eta^e = 5.27 \times 10^3 \text{ms}^{-1}, \quad \chi^e = 3.17 \times 10^3 \text{ms}^{-1}. \]

The software Matlab 7.0.4 has been used to determine the values of energy ratios \( E_i, i = 1, 2 \) and an energy matrix \( E_{ij}, i, j = 1, 2, 3, 4, 5 \) defined in the previous section for different values of incident angle \( (\theta_0) \) ranging from 0 to \( 90^\circ \) for fixed frequency \( \omega = 2 \times \pi \times 100 \text{ Hz} \).

Corresponding to incident P wave, the variations of energy ratios with respect to angle of incidence have been plotted in Figures 5.2-5.9. Similarly, corresponding to incident SV wave, the variations energy ratios with respect to angle of incidence have been plotted in Figures 5.10-5.17. In all figures micropolar thermoelastic diffusion is represented by the word MTED and TED correspond to the thermoelastic diffusion.

**Incident P-wave**

Figure 5.2 exhibits the variation of energy ratio \( E_1 \) with the angle of incidence \( (\theta_0) \). It shows that the values of \( E_1 \) for both cases TED and MTED decrease with the increase in \( \theta_0 \) from \( 0^\circ \) to \( 80^\circ \) and then increase as \( \theta_0 \) increase further. Figure 5.3 depicts the variation of energy ratio \( E_2 \) with \( \theta_0 \) and it indicates that the values of \( E_2 \) increase for smaller values of \( \theta_0 \), whereas for higher values of \( \theta_0 \), the values of \( E_2 \) decrease for both cases TED and MTED. Figure 5.4 depicts the variation of energy ratio \( E_{11} \) with \( \theta_0 \) and it shows that the values of \( E_{11} \) for the case of MTED increase slightly for smaller values of \( \theta_0 \), whereas for the higher values of \( \theta_0 \), the values of \( E_{11} \) decreases, but for the case TED the values of \( E_{11} \) decrease for all values of \( \theta_0 \). Figure 5.5 exhibits the variation of energy ratio \( E_{22} \) with \( \theta_0 \). The similar type of behavior and variation are noticed for \( E_{22} \) as \( E_{11} \) with difference in their magnitude values.
Figure 5.6 depicts the variation of energy ratio $E_{33}$ with $\theta_0$ and it indicates the values of $E_{33}$ for the case of TED show an oscillating behavior in the initial stage, but after that it decrease, whereas for the case of MTED, the values of $E_{33}$ slightly increase for smaller values of $\theta_0$, although for higher values of $\theta_0$, the values of $E_{33}$ decrease. Figure 5.7 depicts the variation of energy ratio $E_{44}$ with $\theta_0$. It shows that the values of $E_{44}$ for both cases TED and MTED increase with the increase $\theta_0$ from $0^\circ$ to $75^\circ$ and then decrease as $\theta_0$ increase further. Figure 5.8 exhibits the variation of energy ratio $E_{55}$ with $\theta_0$ and it indicates the values of $E_{55}$ slightly increase at the initial stage, but after that it decrease. Figure 5.9 depicts the variation of interaction energy ratio $E_{RR}$ with $\theta_0$ and it indicates the values of $E_{RR}$ for the case of MTED decrease, although for the case of TED, the values of $E_{RR}$ decreases for smaller values of $\theta_0$, whereas for higher values of $\theta_0$, the values of $E_{RR}$ increase. If we compare TED and MTED in all figures, we find that the values of $E_1,E_{33},E_{44}$ are higher in TED theory in comparison MTED, but the values $E_2,E_{11},E_{22},E_{RR}$ are more in MTED theory (for higher values of$\theta_0$) in comparison to TED theory.

**Incident SV-wave**

Figure 5.10 represents the variation of energy ratio $E_1$ with $\theta_0$ and it indicates that the values of $E_1$ for both cases TED and MTED increase for smaller values of $\theta_0$, whereas for higher values of $\theta_0$, the values of $E_1$ decrease. Figure 5.11 shows the variation of energy ratio $E_2$ with $\theta_0$. The values of $E_2$ for both cases TED and MTED decrease with the increase in $\theta_0$ from $0^\circ$ to $30^\circ$ and then increase as $\theta_0$ increase further. Figure 5.12 shows that the values of $E_{11}$ for both cases TED and MTED show an oscillatory behavior for initial values of $\theta_0$, whereas for higher values of $\theta_0$, the values of $E_{11}$ decrease. Figure 5.13 exhibits the variation of energy ratio $E_{22}$ with $\theta_0$ and it is noticed that the behavior and variation of $E_{22}$ is similar as $E_{11}$ with difference in their magnitude values. Figure 5.14 exhibits that the values of $E_{33}$ for the case of MTED oscillates, but for the case of TED, the values of $E_{33}$ increase slightly. Figure 5.15 shows the variation of energy ratio $E_{44}$ with $\theta_0$ and it found that the behavior and variation of
\( E_{44} \) is similar as \( E_{33} \) with difference in their magnitude values. Figure 5.16 indicates that the values of \( E_{33} \) increase at the initial stage but after that it decrease. Figure 5.17 represents the variation of \( E_{RR} \) with \( \theta_0 \) and it shows that the values of \( E_{RR} \) for both cases TED and MTED initially decrease, but for higher values of \( \theta_0 \), the values of \( E_{RR} \) decrease for the case of MTED, but for the case of TED, it slightly increases. If we compare TED and MTED in all figure, we find that the values of \( E_1, E_2, E_{33}, E_{44} \) are higher in TED theory in comparison MTED whereas the values \( E_{RR} \) are more in MTED theory (for higher values of \( \theta_0 \)) in comparison to TED theory.

### 5.8 Conclusion

In the present Chapter, the phenomena of reflection and transmission of obliquely incident elastic waves at the interface between an elastic solid half space and a micropolar thermoelastic diffusion solid half space has been investigated. The five waves in micropolar thermoelastic diffusion solid are identified and explained through different wave equations in terms of displacement potentials. Due to the presence of dissipation, the waves in micropolar thermoelastic diffusion medium are considered to be inhomogeneous waves. The energy ratios of different reflected and transmitted waves to that of incident wave are computed numerically and presented graphically with respect to the angle of incidence.

From numerical results, we conclude that the values of \( E_1, E_{33}, E_{44} \) are higher in TED in comparison to MTED, whereas the values \( E_2, E_{11}, E_{22}, E_{RR} \) are more in MTED theory (for higher values of \( \theta_0 \)) in comparison to TED theory in the case of incident P-wave, although for the case of incident SV-wave, we find that the values of \( E_1, E_2, E_{33}, E_{44} \) are higher in TED in comparison to MTED whereas the values \( E_{RR} \) are more in MTED theory (for higher values of \( \theta_0 \)) in comparison to TED. Appreciable micropolarity effect and relaxation times effect are noticed. The sum of all energy ratios of the reflected waves, transmitted wave and interference between transmitted waves for both incident P- and SV- waves is verified to be always unity which ensures the law of conservation of incident energy at the interface.
Figure 5.2 Variation of energy ratio $E_1$ with respect to angle of incident $\theta_0$ for P-wave

Figure 5.3 Variation of energy ratio $E_2$ with respect to angle of incident $\theta_0$ for P-wave
Figure 5.4 Variation of energy ratio $E_{11}$ with respect to angle of incident $\theta_0$ for P-wave

Figure 5.5 Variation of energy ratio $E_{22}$ with respect to angle of incident $\theta_0$ for P-wave
Figure 5.6 Variation of energy ratio $E_{33}$ with respect to angle of incident $\theta_0$ for P-wave

Figure 5.7 Variation of energy ratio $E_{44}$ with respect to angle of incident $\theta_0$ for P-wave
Figure 5.8 Variation of energy ratio $E_{55}$ with respect to angle of incident $\theta_0$ for P-wave

Figure 5.9 Variation of energy ratio $E_{RR}$ with respect to angle of incident $\theta_0$ for P-wave
Figure 5.10 Variation of energy ratio $E_1$ with respect to angle of incident $\theta_0$ for S-wave

Figure 5.11 Variation of energy ratio $E_2$ with respect to angle of incident $\theta_0$ for S-wave
Figure 5.12 Variation of energy ratio $E_{11}$ with respect to angle of incident $\theta_0$ for S-wave

Figure 5.13 Variation of energy ratio $E_{22}$ with respect to angle of incident $\theta_0$ for S-wave
Figure 5.14 Variation of energy ratio $E_{33}$ with respect to angle of incident $\theta_0$ for S-wave

Figure 5.15 Variation of energy ratio $E_{44}$ with respect to angle of incident $\theta_0$ for S-wave
Figure 5.16 Variation of energy ratio $E_{55}$ with respect to angle of incident $\theta_0$ for S-wave

Figure 5.17 Variation of energy ratio $E_{55}$ with respect to angle of incident $\theta_0$ for S-wave