Chapter – I

Introduction
1.1 Historical background of Tribology

The art of lubrication is as old as ancient civilization. The oldest civilization of which we have a clear historical record developed in Mesopotamia about five thousand years ago and there is a clear evidence that they had developed or knew of a number of quite sophisticated tribological devices. Egyptian and Roman Chariot wheels and axels were lubricated to reduce friction with the use of tallow. Many machine elements were widely lubricated with animal fats and lard oils, castor oils were used as a lubricant in aircraft engines. In the second half of the nineteenth century the value of mineral oils as lubricants was first appreciated. As mineral oils meet the requirements; viz viscosity, mechanical stability good thermal stability and resistance to oxidation, admirably these are now used widely in industries. Leonardo da vinci (1452-1519) who has been named as the father of modern tribology, studied an incredible manifold of tribological subtopics such as; function, wear, bearing materials, plain bearings, lubrication systems, gears, screw-jacks and rolling-element bearings 150 years before Amonton’s laws of friction were introduced; he had already recorded them in his manuscripts. Hidden or lost for centuries, Leonardo da Vinci’s manuscripts were read in spain a quarter of a milleniam later. The successors of Leonardo da Vinci in the field of Tribology are guillausne Amontons (1663-1705), John Theophilus Desanguliers (1683-1744). Leonard Euler (1707-1783) and Charles-Augustin Coulomb (1736-1806). These pioneers brought Tribology to a standard and its laws still apply to many engineering problems today. Some of there findings are summarized in the following three laws.

1. The force of friction is directly proportional to the applied load (Amonton’s 1st law)

2. The force of friction is independent of the apparent area of contact (Amonton’s 2nd law)
3. Kinetic friction is independent of the sliding velocity (Coulomb’s Law)

These tree laws were attributed to dry friction only, as it has been well known since ancient times that lubrication modifies the tribological properties significantly. It was Nikolai Pavlovich Petrov and Osborne Reynolds around 1880, who recognized the hydrodynamic nature of lubrication and introduced a theory of fluid film lubrication till today, Reynolds steady state equation of film lubrication is valid for hydrodynamic lubrication.

The scientific study of lubrication began with Reyleigh together with stokes discussed the feasibility of theoretical treatment of film lubrication. The understanding of hydrodynamic lubrication began with the classic experiments of Tower (1884), in which the existence of a film was detected from measurements of pressure within the lubricant, and of Pedroff (1883), who reached the same conclusion from friction measurements. Reynold’s (1886) closely followed this work, in his analytical paper he used a reduced form of the Nevier-Stoke’s equation in association with the continuity equation to generate a second-order partial differential equation for the pressure in the narrow, conversing gap between bearing surfaces. This pressure enables a load to be transmitted between the surfaces with extremely low friction, since the surfaces are completely separated by a fluid film.

1.2 Types of lubrication

The process of reducing wear and heat between contacting surfaces in relative motion is known as lubrication theory. While wear and heat cannot be completely eliminated but they can be reduced to negligible or acceptable levels by using the lubricants. The effect of heat and wear can be minimized by reducing the coefficient of friction between the contacting surfaces as they are associated with friction. Lubrication is also used to reduce oxidation and prevent rust; to provide insulation in transformer
application; to transmit mechanical power in hydraulic fluid power applications; and to seal against dust, dirt, and water. Mainly there are four types of lubrication viz; hydrodynamic, Aerodynamic, Elstohydrodynamic and boundary lubrication.

1.2.1 Hydrodynamic lubrication

In heavily loaded bearings such as thrust bearings and horizontal journal bearings, the fluid’s viscosity alone is not sufficient to maintain a film between the moving surfaces. In these bearings higher fluid pressure are required to support the load until the fluid film is established. If this pressure is supplied by an outside source, it is called hydrostatic lubrication. If the pressure is generated internally, that is, within the bearing by dynamic action, it is referred to as hydrodynamic lubrication. In hydrodynamic, a fluid wedge is formed by the relative surface motion of the journals or the thrust runners over their respective bearing surfaces. This type of lubricating action is similar to a speedboat operating on water. When the boat is not moving, it rests on the supporting water surface. As the boat begins to move, it meets a certain amount of resistance or opposing force due to viscosity of the water. This causes the leading edge of the boat to lift slightly and allows a small amount of water too come between it and supporting water surface. As the boat’s velocity increases, the wedge –shaped water film increases in thickness until a constant velocity is attained. When the velocity is constant, water entering under the leading edge equals the amount passing outward from the trailing edge. For the boat to remain above the supporting surface there must be an upward pressure that equals the load. The principle of hydrodynamic lubrication can also be applied to a more practical example related to thrust bearings used in the hydropower industry.
1.2.2 Aerodynamic lubrication

Aerodynamic lubrication is a recent extension of the theory of hydrodynamic lubrication.

The theory is the same, in that converging wedge of high pressure is formed between two surfaces, supporting one surface from coming in to contact with the other surface. Instead of using the thin fluid lubricant, a gas of 1000 times less viscosity is used. The distance of separation is minute, requiring close to perfectly smooth surfaces. Besides the surfaces having to be virtually free of defects, using aerodynamic lubrication requires very high speeds, and low loads.

1.2.3 Elstohydrodynamic lubrication

Elstohydrodynamic lubrication can be defined as a form of hydrodynamic lubrication where the elastic deformation of the contacting bodies and the changes of viscosity with pressure play the fundamental roles. The influence of elasticity is not limited to second order changes in load capacity or friction as described for pivoted pad and journal bearings. Instead, the deformation of the bodies has to be included in the basic model of elstohydrodynamic lubrication. The same refers to the changes in viscosity due to pressure.

1.2.4 Boundary lubrication

Boundary lubrication is the most common type of lubrication in day-to-day usage because, in it finds its applicability where hydrodynamic and elastodynamic lubrication fails. When a complete fluid film does not develop between potentially rubbing surfaces, the film thickness may be reduced to permit momentary dry contact between wear surface with high points or asperities. This condition is characteristic of boundary lubrication.
Boundary lubrication occurs whenever any of the essential factors that influence formation of a full fluid film are missing.

1.3 Classification of fluids

Fluid is a substance which deforms continuously, without limit, under the action of internal and external forces. In other words, a fluid is a substance which cannot resist a shear stress without moving as can a solid. Fluids are usually classified as liquids and gases. A liquid has intermolecular forces which hold it together so that it has volume but no definite shape. A gas on the other hand has molecules in motion which collide with each other tending to disperse so that a gas has no definite shape or volume. Fluids are classified into following two categories, ideal fluids or in viscid fluids and real fluids or viscous fluids.

1.3.1 Newtonian fluids

The viscosity $\mu$ is measured by the slope of stress-shearing rate curve. For natural fluids like, water, air, oil and so on viscosity does not vary with rate of strain. That is fluids with constant viscosity are known as Newtonian fluids. For Newtonian fluids shear is linearly proportional to the rate of strain i.e.

$$\tau = \mu \gamma$$

Where $\tau$ is the shear stress, $\mu$ is the viscosity of fluid and $\gamma$ is the rate of shearing strain. Newtonian behavior is exhibited by fluids in which the dissipation of viscous energy is due to the collision of comparatively small molecular spaces. All gases and liquids and solutions of low molecular weight come into this category.
1.3.2 Non-Newtonian fluids

Non-Newtonian fluids are that for which the flow curve is not linear, i.e. the viscosity of a non-Newtonian fluid is not constant at a given temperature and pressure but depends on the other factor such as the rate of shear in the fluid. Examples of non-Newtonian fluids are pastes, printers ink, condensed milk, molten rubber, molasses, and high polymer solution and so on. The non-Newtonian fluids for which the flow curve is not linear may be classified into three broad categories (Wilkinson, 1960; Harris, 1976).

1.4 Viscosity variation

Viscosity is a measure of the resistance of a fluid to flow. When the temperature of a liquid is changed, the distance between molecules changes and this in tern affects the viscosity. Liquids with low coefficients of expansion will in general have lower viscosity-temperature coefficients than those which have high coefficients of expansion.

Faust proposed a theory in 1914 that the viscosity of a given liquid is a function of density alone, regardless of temperature and pressure. Bridman later proved that this theory is an over simplification. He made viscosity measurements on a number of liquids over a range of temperatures and pressures, and found that change in density does not account for all of the effect of temperature on viscosity, although it is responsible for a substantial part of the total effect. The viscosity of lubricants increases markedly with increasing pressure. At the pressures existing in the lubricant film of hydrodynamic bearings, the viscosity of the lubricant may many times greater than its viscosity as measured at atmospheric pressure. This property of lubricants undoubtedly has an influence on bearing performance characteristics such as load-carrying capacity, friction and temperature rise.
There is no simple method of measuring viscosity at high pressure. A program of measurement to define the pressure-viscosity-temperature properties of a single lubricant assumes the proportions of a research program rather than of a routine physical property measurement. Consequently pressure-viscosity-temperature that are relatively scarce and the effect of pressure-viscosity properties on bearing performance is not well understood as may be desirable.

The variation in viscosity with temperature is important in many practical applications, where lubricants are required to function over a wide range of temperature Freeman (1962). The formulae proposed for defining the viscosity-temperature relationship are purely empirical, and for accurate calculations the lubrication engineers require experimental data. However, a viscosity-temperature relationship can be replaced by a viscosity film thickness relationship as it has been verified experimentally that, the highest temperature occurs in zones where the film thickness is lowest Tipei (1962).

When the viscosity $\mu_1$ at $h = h_1$ (oil inlet condition) is known, then

$$\mu = \mu_1 (H/h_1)^{\theta}$$

Where $\theta$, usually lies between 0 and 1 (according to the nature of the lubricant) Sinha et al. (1981) studied the effect of viscosity variation in journal bearings lubricated with micropolar fluids. Effect of viscosity variation on the squeeze film performance of a narrow hydrodynamic journal bearing operating with couple stress fluid is studied by JayaChandra et al. (2007). In this paper they analyzed that, the viscosity variation factor decreases the load carrying capacity and squeeze film time. Effect of viscosity variation on the micropolar fluid squeeze film lubrication of a short journal bearing is studied by Naduvinamani and Archana (2013). Siddangouda et al. (2013) studied combined effects of surface roughness and viscosity variation due to additives on long journal bearing. He
observed that, the combined effect is to increase the load carrying capacity and to decrease the coefficient of friction, which improves the performance of the bearing.

1.5 Micropolar fluids

Number of theories has been proposed to explain the peculiar behavior of fluids which contain microstructures such as additives and suspension of granular matter. The theory of micropolar fluids introduced by Eringen (1964) deals with a class of fluids which exhibits certain microscopic effects arising from the local structures and micro motion of fluid elements. These fluids can support stress moments and body moments and are influenced by the spin inertia. A subclass of these fluids is the micropolar fluids which exhibits the microrotational effects and microrotational inertia. Eringen’s (1964) micropolar fluid theory defines the ration vector called micro rotation vector setting up of stress-strain rate constitutive equations. The study of micropolar fluids has been received considerable attention due to their applications in a number of processes that occur in industries such as extrusion of polymer fluids, solidification of liquid crystal, cooling of metallic plate in a bath, Animal blood exotic lubricants and colloidal and suspension solution. In the study of all these problems, the classical Navier-Stokes theory is inadequate. As the micropolar fluid theory is a subclass of micro fluid theory and is obtained by imposing the skew symmetric properties of the gyration tensor in addition to a condition of micro isotropy.

A new generalization of the Reynolds equation for a micropolar fluid and its application to bearing theory is presented by Bessonov (1994). Khonsari (1994) studied the effect of viscous dissipation on the lubrication characteristics of micropolar fluids and shown that the heat generation due to viscous dissipation plays an important role on the load carrying capacity of a journal bearing lubricated with micropolar fluids. Lin (1996) analyzed the
hydrodynamic lubrication of journal bearings including micropolar lubricants and the three dimensional irregularities. On the conical whirls instability of hydrodynamic journal bearing lubricated with micropolar fluids is presented by Das et al. (2001). It is found that, for any micropolar lubrication condition, the bearing is always stable as the ratio of the moment of inertia approaches the value of conical whirl ratio. Static characteristics of a circular journal bearing operating with micropolar lubricant considering the effect of deformation of bearing liner were computed by Nair et al. (2004). They have shown that the static characteristics are greatly affected by the volume concentration of additives in the lubricant. Das et al. (2005) have analyzed the linear stability analysis of hydrodynamic journal bearing under micropolar lubrication. Naduvinamani and Marali (2007) presented the dynamic Reynolds equation for micropolar fluids and the analysis of plane inclined slider bearings with squeezing effect and have shown that the micropolar fluids provide an improved characteristics for both steady state and the dynamic stiffness and damping characteristics. It is found that, the maximum steady load carrying capacity is a function of the coupling parameter. Naduvinamani and Siddanagouda (2008) studied the porous inclined stepped composite bearings with micropolar fluid. It is observed that the micropolar fluid lubricants provide an increased load carrying capacity and decreased coefficient of friction as compared to the corresponding Newtonian case. Naduvinamani and Santosh (2009) analyzed theoretically micropolar fluid squeeze film lubrication of short partial porous journal bearing. Rahmatabadi et al. (2010) showed that the significant enhancement in static performance for different configuration of non-circular bearings by considering micropolar lubrication. Sanyam and Rattan (2010) studied the micropolar lubricant effects on the performance of a two-lobe bearing with pressure dam, this paper shows that two-lobe pressure dam bearing is superior to two lobe bearing. Naduvinamani and Santosh (2011) studied the micropolar fluid squeeze film lubrication of finite porous
journal bearing. It is observed that, the micropolar fluid effect significantly increases the squeeze film pressure and the load carrying capacity as compared to the corresponding Newtonian case. Analyzing micropolar lubrication in non-circular hybrid journal bearing is analyzed by Rohit and Suresh (2013) in this paper they suggests that a non-circular bearing configuration operating with a micropolar lubricant offers better performance as compared to operation with a Newtonian lubricant. Naduvinamani and savitramma (2014) studied the micropolar fluid squeeze film lubrication between rough anisotropic poro-elastic rectangular plates- a special reference to synovial joint lubrication. In present thesis the viscosity variation on the squeeze film lubrication with micropolar fluid.

1.6 Rabinowitsch fluids

However, the studies of hydrodynamic lubrication behavior focus on the assumption that the performance of bearings lubricated with a Newtonian viscous fluid. A lubricant is a substance interposed between two surfaces in relative motion for the purpose of reducing the friction and wear between them. Lubricant provides a protective film which allows for two touching surfaces to be separated and “smoothed,” thus lessening the friction between them and correspondingly less heat generation in the machine, thereby keeping the working temperature of machine parts within safe operating limits. But in reality, it is found that, the Newtonian fluid constitutive approximation cannot satisfy engineering demands of modern lubricants. Modern equipment must be lubricated in order to prolong its lifetime. Nowadays, the use of small amount of additives to the lubricant is becoming of great interest. The flow behavior of a Newtonian lubricant blended with various additives cannot be accurately described by the classical continuum theory. The viscosity of these lubricants exhibits a non-linear relationship between the shear stress and the shear strain rate.
Many researchers have investigated the influence of non-Newtonian properties upon different lubrication problems based on Rabinowitsch fluid model. Hsu and Saibel (1965) analyzed slider bearing performance with a non-Newtonian lubricant. They concluded that in the case of a pseudoplastic fluid the pressure distribution is lowered over the Newtonian case, and consequently the load carrying capacity is reduced. On the other hand this appears to be more than compensated for by the increase in flow rate and the decrease in the friction force. Non-Newtonian Flow in Infinite-Length Full Journal Bearing was analyzed by Hsu (1967). He concluded that the pseudoplasticity and dilatants nature of the fluid provide a significant influence on the pressure distribution, flow rate, frictional force, load carrying capacity, attitude angle, and frictional coefficient. Hydrodynamic lubrication of journal bearings by pseudoplastic lubricants was studied by Wada and Hayashi (1971). They found that the pressure distribution of pseudoplastic fluids decreases below that of Newtonian fluids whose viscosity is equal to the initial viscosity of pseudoplastic fluids and the load capacity and the frictional force also decrease. Non-Newtonian effects on the static characteristics of one-dimensional slider bearings in the inertial flow regime were studied by Hashimoto (1994). He observed that the non-Newtonian effects and fluid inertia effects provide significant influences upon the static characteristics of one-dimensional, high-speed slider bearings. Variation principle for non-Newtonian lubrication: Rabinowitsch fluid model was analyzed by He (2004). Singh et.al (2011) presented on the steady performance of hydrostatic thrust bearing: Rabinowitsch fluid model. They found that the results so obtained are compared and found to be in good agreement with the earlier theoretical and practical results of Dowson, and the effect of viscosity index improver is analyzed. Effects of inertia in the steady state pressurized flow of a non-Newtonian fluid between two curvilinear surfaces of revolution: Rabinowitsch fluid model was studied by Singh et. al (2011). They
observed that the pseudoplastic effect along with the effect of rotational inertia on the pressure distribution, frictional torque and fluid flow rate of externally pressurized flow in narrow clearance between two curvilinear surfaces of revolution. Lin (2012) analyzed non-Newtonian squeeze film characteristics between parallel annular disks: Rabinowitsch fluid model. He found that the influences of dilatants properties provide an increase in the load-carrying capacity, and therefore lengthen the response time; however, the effects of pseudoplastic characteristics yield a reversed trend as compared to the case of Newtonian lubricants. Further results have been presented through the variation of the nonlinear non-Newtonian parameter and the radius ratio. Singh and Gupta (2012) presented non-Newtonian effects on the squeeze film characteristics between a sphere and a flat plate: Rabinowitsch model. They concluded that the film pressure distribution, load carrying capacity, and squeezing time characteristics show a significant variation with the non-Newtonian pseudoplastic and dilatants behavior of the fluids. On the performance of Pivoted Curved Slider Bearings: Rabinowitsch Fluid Model was analyzed by Singh et. al (2012). They found that the steady state bearing performance characteristics i.e., steady state film pressure, load carrying capacity and centre of pressure, as well as dynamic stiffness and damping characteristics vary significantly with the non-Newtonian behavior of the fluid consistent with the real nature of the problem. Singh et. al (2012) analyzed on the steady performance of annular hydrostatic thrust bearing: Rabinowitsch fluid model. They observed that, in the limiting case in which there is an absence of pseudoplasticity, the results are compared with the pre-established Newtonian lubricants and are found to be in good agreement. On the squeeze film characteristics between a long cylinder and a flat plate: Rabinowitsch model was presented by Singh et. al (2013). They concluded that the pressure distribution, load-carrying capacity and time-height relationship under squeezing characteristics show significant variation with the non-Newtonian
pseudoplastic and dilatants behavior of the fluids. Effect of non-Newtonian Rabinowitsch fluids in wide parallel rectangular squeeze-film plates were studied by Lin et. al (2013). They found that, comparing with the Newtonian-lubricant parallel plates, the effects of non-Newtonian cubic-stress flow rheology provide significant influences upon the squeeze film characteristics. It is shown that the non-Newtonian pseudoplastic behavior reduces the load capacity and the response time; however, the effects of non-Newtonian dilatants lubricant provide an increase in the load-carrying capacity and therefore lengthen the response time of parallel squeeze-film plates.

1.7 Surface Roughness

Roughness is one of the most important surface topographic characterizations, which intuitively refers to the unevenness or irregularity of a texture. It gives an idea of how smooth the surface is at a certain length scale. Roughness is dependent on the vertical and horizontal resolution of the measuring instruments. It is also a function of working length scale.

The study of the effects of surface roughness on hydrodynamic lubrication of various bearing systems has been a subject of growing interest, mainly because, in practice, most of the bearing surfaces are rough. The aspect ratio and the absolute height of the asperities and valleys observed under microscope vary greatly, depending on material properties and on the method of surface preparation. In general, the height of roughness asperities is of the same order as the mean separation in a lubricated contact.

Stochastic models for hydrodynamic lubrication of rough surfaces were studied by Christensen (1969-70). On the basis of stochastic theory, he developed two different forms of Reynolds-type equation corresponding to two different types of surface
roughness. It is shown that the mathematical form of these equations is similar but not identical to the form of the Reynolds equation governing the behavior of smooth, deterministic bearing surfaces. It is shown that surface roughness may considerably influence the operating characteristics of bearings and that the direction of the influence depends upon the type of roughness assumed. The effects are not, however, critically dependent upon the detailed form of the distribution function of the roughness heights. Christensen (1971) presented some aspects of the functional influence of surface roughness in lubrication. He displayed that, surface roughness has a considerable effect on the functional characteristics of a bearing operating in the hydrodynamic, and, especially, in the mixed lubrication regime. Waviness and roughness in hydrodynamic lubrication was presented by Tonder and Christensen (1972). They observed that the corrugation wavelength is a major factor, pressure ripples vanishing with increasing corrugation density. It is further shown that at the same time, the load-carrying capacity tends towards that predicted by the authors' statistical roughness theory, the analysis thus constituting a numerical proof of the mathematical soundness of that theory. An analysis of the squeeze film between porous rectangular plates including the surface roughness effects was studied by Prakash and Tiwari (1982). They observed that the nominal geometry as characterized by the aspect ratio of the plates has a profound effect on the system. Guha (1993) presented the analysis of dynamic characteristics of hydrodynamic journal bearings with isotropic roughness effects. The effect of roughness on the behavior of squeeze film in a spherical bearing is studied by Gupta and Deheri (1996). A note on squeeze film between rough anisotropic porous rectangular plates was analyzed by Bujurke and Naduvinamani (1998). They found that the loci of maximum load are more sensitive to the anisotropic permeability than the roughness parameter. Gururajan and Prakash (1999) presented surface roughness effects in infinitely long porous journal
bearings. Effect of surface roughness in a narrow porous journal bearing was studied by Gururajan and Prakash (2000). It is shown that the results are significantly different than those for the case of an infinitely long journal bearing. The surface roughness effects on the oscillating squeeze film behavior of long partial journal bearings are studied by Lin et al. (2002) on the basis of Christensen Stochastic model and found that the effect of circumferential roughness provides a reduction in the mean bearing eccentricity ratio as compared to smooth bearing case where as the longitudinal roughness structure results in a reverse trend. Naduvinanmani et al. (2003) analyzed effect of surface roughness on characteristics of couplestress squeeze film between anisotropic porous rectangular plates. They found that the surface roughness effects are more pronounced for couplestress fluids as compared to the Newtonian fluids. Chiang et al. (2004) presented linear stability analysis of a rough short journal bearing lubricated with non-Newtonian fluids. Hsiu-Lu Chiang et al. (2005) studied that surface roughness effects on the dynamic characteristics of finite slider bearings. They observed that when compared to the smooth bearing lubricated with couple stress fluid, a decrease in the threshold speed is found in the case of transverse roughness. Bujurke and Kudenatti (2006) presented surface roughness effects on squeeze film poroelastic bearings. Finite-difference-based multigrid method is used for the solution of Reynolds equation. Surface roughness effects on thermo-hydrodynamic lubrication of journal bearings lubricated with bubbly oil is studied by E1-Butch and E1-Tayeb (2006). The method has greatest advantage of minimizing the errors using correction schemes in obtaining accurate solution as grid size tends to zero. The influences of roughness and elasticity on bearing characteristics are discussed. Bujurke et al. (2007) studied the effect of surface roughness on squeeze film poroelastic bearings with special reference to synovial joints. Bujurke et al. (2007) investigated the wavelet-multigrid analysis of squeeze film characteristics of poroelastic bearings. They
found that the poroelastic bearings with couple-stress fluid as lubricant provide enhancement in pressure and ensure the increased load carrying capacity compared with viscous fluids. This may be one of the reasons in the efficient lubrication and proper functioning of synovial joints. Naduvinamani and Biradar (2008) analyzed the surface roughness effects on the static and dynamic behavior of squeeze film lubrication of short journal bearing with micropolar fluids. Influence of roughness parameters on coefficient of friction under lubricated conditions is analyzed by Pradeep et al. (2008). Numerical solution of finite modified Reynolds equation for couple stress squeeze film lubrication of porous journal bearings was analyzed by Naduvinamani and Patil (2009). They found that the applied load is considered as a sinusoidal function of time to simulate the bearings operating under cyclic loads. Under a cyclic load, the effect of couple stress is to reduce the velocity of the journal centre and to increase the minimum permissible height of the squeeze film. Naduvinamani and Kashinath (2010) analyzed hydrodynamic analysis of rough curved pivoted porous slider bearings with couple stress fluid. They found that the improved performance due to the couple stresses and the presence of negatively skewed surface roughness. However, the presence of porous facing and positively skewed surface roughness affects the performance of the pivoted porous slider bearing. Elsharkawy and Fadhalah (2011) studied squeeze film characteristics between a sphere and a rough porous flat plate with micropolar fluids. They observed that, excessive permeability of the porous layer causes a significant drop in the squeeze film characteristics and minimizes the effect of surface roughness. For the case of limited or no permeability, the azimuthally roughness is found to increase the load-carrying capacity and squeeze time, whereas the reverse results are obtained for the case of radial roughness. Combined effects of MHD and surface roughness on couple-stress squeeze film lubrication between porous circular stepped plates are analyzed by Naduvinamani et al. (2012). Walicka (2012) studied that,
inertia effects in porous squeeze film biobearing with rough surfaces lubricated by a power-law fluid. It is shown that the inertia effects, power-law exponents, and surface roughness influence the biobearing performance considerably. Squeeze film lubrication between rough poroelastic rectangular plates with micropolar fluid: A special reference to the study of Synovial joint lubrication was studied by Naiduvinamani and Savitramma (2013). They observed that the effect of surface roughness has considerable effects on lubrication mechanism of synovial joints.

1.8. Basic Equations

1.8.1. Micropolar fluids

The field equations for micropolar fluids in vectorial form are (Eringen, 1966)

Conservation of mass

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \]  \hspace{1cm} (1.8.1.1)

Conservation of linear momentum

\[ (\lambda + 2\mu + k)\nabla \cdot V - (\mu + k)\nabla \times V + k\nabla \times V - \nabla \pi + \rho f = \rho \left[ \frac{\partial V}{\partial t} - \nabla \times (\nabla \times V) + \nabla \nabla \left( \frac{1}{2} (\nabla \nabla (V^2)) \right) \right] \]  \hspace{1cm} (1.8.1.2)

Conservation of angular momentum

\[ (\alpha + \beta + \gamma)\nabla (\nabla \cdot v) - \gamma \nabla \times (\nabla \times v) + k\nabla \times V - 2kV + \rho l = \rho \dot{V} \]  \hspace{1cm} (1.8.1.3)

The first expression is the principle of conservation of mass and the others are the conservation of linear and angular momentum. A superimposed dot indicates material differentiation. For an incompressible fluid \( \rho = \text{const} \), \( \nabla \cdot v = 0 \) and \( \pi \) is replaced by an unknown pressure to be determined by the boundary conditions.
For the three dimensional steady motion of an incompressible micropolar fluid under the assumption of negligible body forces and body couples, the field equations (1.8.1.1)-(1.8.1.3) reduce to

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1.8.1.4)
\]

\[
\frac{1}{2} (2\mu + \chi) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \chi \left( \frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z} \right) - \frac{\partial p}{\partial x} = \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \quad (1.8.1.5)
\]

\[
\frac{1}{2} (2\mu + \chi) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \chi \left( \frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x} \right) - \frac{\partial p}{\partial y} = \rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) \quad (1.8.1.6)
\]

\[
\frac{1}{2} (2\mu + \chi) \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \chi \left( \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \right) - \frac{\partial p}{\partial z} = \rho \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) \quad (1.8.1.7)
\]

\[
\gamma \left( \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right) + \chi \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) - 2\chi v_1 = \rho \left( u \frac{\partial v_1}{\partial x} + v \frac{\partial v_1}{\partial y} + w \frac{\partial v_1}{\partial z} \right) \quad (1.8.1.8)
\]

\[
\gamma \left( \frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2} + \frac{\partial^2 v_2}{\partial z^2} \right) + \chi \left( \frac{\partial w}{\partial z} - \frac{\partial v}{\partial x} \right) - 2\chi v_2 = \rho \left( u \frac{\partial v_2}{\partial x} + v \frac{\partial v_2}{\partial y} + w \frac{\partial v_2}{\partial z} \right) \quad (1.8.1.9)
\]

\[
\gamma \left( \frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial y^2} + \frac{\partial^2 v_3}{\partial z^2} \right) + \chi \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) - 2\chi v_3 = \rho \left( u \frac{\partial v_3}{\partial x} + v \frac{\partial v_3}{\partial y} + w \frac{\partial v_3}{\partial z} \right) \quad (1.8.1.10)
\]

The constitutive equations for the stress tensor \( t_{kl} \) and the couple stress tensor \( m_{kl} \) are given in Cartesian co-ordinates as
\[ t_{kl} = (-\pi + \lambda v_{r,r}) \delta_{kl} + \mu \left( v_{k,l} + v_{l,k} \right) + k \left( V_{r,k} - \epsilon_{klr} v_r \right) \]  \hspace{1cm} (1.8.1.11)

\[ m_{kl} = \alpha v_{r,r} \delta_{kl} + \beta v_{k,l} + \gamma v_{l,k} \]  \hspace{1cm} (1.8.1.12)

where \( \delta_{kl} \) is the Kronecker delta and \( \epsilon_{klr} \) is the alternating symbol. An index followed by a comma represents partial differentiation with respect the space variable \( x_k \).

The lubrication films to be analyzed are assumed to comply with usual assumptions of hydrodynamic lubrication (Pinkus and Sternlicht, 1961) i.e., the flow is laminar, body forces are neglected, no slip on the bearing surface and film is sufficiently thin in comparison with the length and the span of the bearing equations (1.8.1.1) to (1.8.1.12) are simplified by these assumptions, the velocity components and micro-rotational components reduce to

Conservation of linear momentum:

\[ \left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v_3}{\partial y} - \frac{\partial p}{\partial x} = 0 \]  \hspace{1cm} (1.8.1.13)

\[ \left( \mu + \frac{\chi}{2} \right) \frac{\partial^2 w}{\partial y^2} - \chi \frac{\partial v_1}{\partial y} - \frac{\partial p}{\partial z} = 0 . \]  \hspace{1cm} (1.8.1.14)

Conservation of angular momentum:

\[ \gamma \frac{\partial^2 v_1}{\partial y^2} - 2 \chi v_1 + \chi \frac{\partial w}{\partial y} = 0 , \]  \hspace{1cm} (1.8.1.15)

\[ \gamma \frac{\partial^2 v_3}{\partial y^2} - 2 \chi v_3 - \chi \frac{\partial u}{\partial y} = 0 \]  \hspace{1cm} (1.8.1.16)

\[ \frac{\partial p}{\partial y} = 0 \]  \hspace{1cm} (1.8.1.17)
Conservation of mass:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1.8.1.18) \]

The four differential equations (1.8.1.13)-(1.8.1.16) form two systems of simultaneous equations. One consists of equations (1.8.1.13) and (1.8.1.16) and the other of equations (1.8.1.14) and (1.8.1.15). Solution of these equations (1.8.1.13) and (1.8.1.16) gives expressions for \( u \) and \( v_3 \) and the solution of equations (1.8.1.14) and (1.8.1.15) yields the expression for \( w \) and \( v_1 \). The second system becomes identical with the first system if \( v_3 \) is replaced by \( v_1 \). Hence it is necessary to solve only the first system for \( u \) and \( v_3 \) and the expression for \( w \) and \( v_1 \) can be written in a similar way.

The boundary conditions for the velocity and the micro rotation velocity are

(i) At the upper surface \((y = h)\):

\[ u = U_{11}, \quad w = U_{12}, \quad v = v_h, \quad v_1 = 0, v_3 = 0 \quad (1.8.1.19) \]

(ii) At the lower surface \((y = 0)\):

\[ u = U_{21}, \quad w = U_{22}, \quad v = v_0, \quad v_1 = 0, v_3 = 0 \quad (1.8.1.20) \]

The solution of equations (1.8.1.13)-(1.8.1.16) subject to boundary conditions given in

(1.8.1.19) \quad \text{and} \quad (1.8.1.20) \quad \text{is obtained in}

\[ u = \frac{1}{\mu} \left( \frac{y^2}{2} \frac{\partial p}{\partial x} + D_{11} y \right) - \frac{2N^2}{m} \times \left[ D_{21} \sinh (my) + D_{31} \cosh (my) \right] + D_{41} \quad (1.8.1.21) \]

\[ w = \frac{1}{\mu} \left( \frac{y^2}{2} \frac{\partial p}{\partial z} + D_{12} y \right) - \frac{2N^2}{m} \times \left[ D_{22} \sinh (my) + D_{32} \cosh (my) \right] + D_{42} \quad (1.8.1.22) \]
\[ v_1 = \frac{1}{2\mu} \left( y \frac{\partial p}{\partial z} + D_{12} \right) - \left[ D_{22} \cosh(my) + D_{32} \sinh(my) \right] \quad (1.8.1.23) \]

\[ v_3 = D_{21} \cosh(my) + D_{31} \sinh(my) - \frac{1}{2\mu} \left( y \frac{\partial p}{\partial x} + D_{11} \right) \quad (1.8.1.24) \]

where \( D_{ij} (i=1,2,3,4 \text{ and } j=1,2) \) are constants and given by

\[ D_{ij} = \frac{D_{ij}}{2\mu}, \]

\[ D_{2j} = \frac{D_{2j}}{2\mu}, \]

\[ D_{3j} = \frac{1}{\mu} \left[ (U_{2j} - U_{1j}) \frac{1 - \cosh(mh)}{2} + \frac{h}{2\mu} \frac{\partial p}{\partial x_j} \left( \frac{h}{2} \left( \cosh(mh) - 1 \right) + \frac{N^2}{m} \sinh(mh) \right) \right] \frac{1}{D_3}, \]

\[ D_{4j} = U_{ij} + \frac{2N^2}{m} \times D_{3j}, \]

\[ D_5 = \frac{h}{\mu} \left[ \sinh(mh) - \frac{2N^2}{m} \left( \cosh(mh) - 1 \right) \right], \]

in which \( m = \frac{N}{l}, \quad x_1 = x, \quad x_2 = z. \)

The flow volume rates \( q_x \) and \( q_y \) along the \( x \) and \( z \)-axis respectively are given by

\[ q_x = \int_0^h \int_0^h u dy \quad \text{and} \quad q_z = \int_0^h \int_0^h w dy. \]

Substituting the expressions for \( u \) and \( w \) from Eqns. (1.8.1.21) and (1.8.1.22) into the above integrals and simplifying, we get

\[ q_x = \frac{h}{2} (U_{11} + U_{21}) - \frac{f(N,l,h)}{12} \mu \frac{\partial p}{\partial x}, \quad (1.8.1.26) \]
\[ q_z = \frac{h}{2} \left( U_{12} + U_{22} \right) - \frac{f \left( N, h, l \right)}{12} \mu \frac{\partial p}{\partial z}. \]  

(1.8.1.27)

Where \( f \left( N, h, l \right) = h^3 + 12l^2 h - 6Nh \coth \left( \frac{Nh}{2l} \right) \)

Integrating the equation of continuity (1.8.1.18) across the film and using the equations (1.8.1.21) and (1.8.1.22) together with equations (1.8.1.19) and (1.8.2.20), we get the generalized Reynolds equation for micropolar fluid in three dimensional form as

\[
\frac{\partial}{\partial x} \left( f \left( N, h, l \right) \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( f \left( N, h, l \right) \frac{\partial p}{\partial z} \right) =
\]

\[
6 \frac{\partial}{\partial x} \left( U_{11} - U_{22} \right) h + 12 h \frac{\partial}{\partial z} \left( U_{12} + U_{22} \right) + 12 \left( v_h - v_0 - U_{11} \frac{\partial h}{\partial x} - U_{22} \frac{\partial h}{\partial z} \right)
\]

Where \( f \left( N, h, l \right) = h^3 + 12l^2 h - 6Nh \coth \left( \frac{Nh}{2l} \right) \), \( v_h \) and \( v_0 \) are normal velocity components of the surface at \( y = h \) and \( y = 0 \) respectively.

1.8.2 Rabinowitsch Fluid

The Rabinowitsch fluid (cubic equation) model describes the non-linear relationship between the shear stress and the shear strain rate for the non-Newtonian lubricants. In this model the following relationship holds for one-dimensional flow.

\[
\tau_{xy} + k^* \tau_{xx}^3 = \mu_0 \frac{\partial u}{\partial x}
\]

(1.8.2.1)

Where \( \mu_0 \) denotes the zero shear rate viscosity and is equivalent to the viscosity of Newtonian fluids, and \( k^* \) represents a non-linear factor accounting for non-Newtonian effects. The cubic equation model can be applied to dilatants fluids for \( k^* < 0 \), Newtonian fluids for \( k^* = 0 \) and pseudoplastic fluids for \( k^* > 0 \), respectively.