Chapter – IV

Effect of Pressure-Dependent Viscosity on the Squeeze Film Characteristics of Micropolar Fluid in Convex Curved Plates.
Part of this chapter has been communicated to the journal “Tribology Materials, Surfaces and Interface” (2014)

4.1 Introduction

The theory of micropolar fluids introduced by Eringen (1966) deals with a class of fluid which exhibits certain microscopic effects arising from the local structures and micro motion of fluid elements. These fluids can support stress moments and body moments and are influenced by the spin inertia. Eringen’s micropolar fluid theory defines the rotation vector called micro rotation vector setting up of stress-strain rate constitutive equations. The study of micropolar fluids has been received considerable attention due to their applications in a number of processes that occur in industries such as extrusion of polymer fluids, solidification of liquid crystal, cooling of metallic plate in a bath, animal blood exotic lubricants and colloidal and suspension solution. In the study of all these problems, the classical Navier-Stokes theory is inadequate.

Several researchers Allen et. al. (1971), Verma et. al. (1979), Das et al. (2004), Naduvinamani et. al. (2009), Naduvinamani et. al. (2010), have studied the micropolar fluid lubrication theory which is characterized by the presence of particles (microstructures) suspended in the fluid.

In the literature, the squeeze film behaviors between curved plates have been analyzed by assuming the lubricant with an iso-viscous property, such as the rectangular sine-curved films by Hay (1963), the circular exponential-curved films by Murti (1975), the sphere-socket films by Lin (1996), the cylinder plate films by Lin et al. (2004). The effect of velocity-slip and viscosity variation for Lubrication of Roller Bearing has been studied by Rao and Prasad (2003) and the sphere-plate film was studied by Lin, J.R. (2000). On the other hand, an experimental study carried out by Barus (1893) shows that, under the isothermal condition the viscosity of the lubricant vary with the film pressure.
He suggested that the lubricant viscosity $\mu$ grows with the pressure described by an exponential equation.

$$\mu = \mu_0 e^{ap}$$

(4.1.1)

where $p$ is the film pressure, $a$ is the pressure-dependent constant and $\mu_0$ is the lubricant viscosity at atmospheric pressure. Since the squeeze film pressure is generally increasing with decreasing film thickness, the pressure-dependent viscosity (PDV) should be considered especially for upper plate achieves a smaller squeeze film height. Therefore, a further study between curved squeeze film surfaces including the effects of PDV in equation (4.1.1) is of interest. In the present study, a squeeze film mechanism of convex curved plates of a cosine form with micropolar fluids is proposed. Comparing with the case of iso-viscous lubricants, the effects of PDV on the squeeze film characteristics are presented and discussed through the variation of the PDV parameter, the coupling number, characteristic length, amplitude ratio and the non-dimensional minimum film thickness of the cosine-form convex plates.

4.2 Mathematical Formulation of the problem

Figure 4.1 shows the squeeze film geometry between convex curved plates with a cosine form. The film thickness $h$ can be generated by the form of a cosine function.

$$h = h_m + d[1 - \cos(\frac{\pi}{L} x)]$$

(4.2.1)

in this equation, $L$ is the length of the plate, $d$ is the amplitude of the cosine function, and $h_m$ is the minimum film thickness. The point $x = 0$ is at the central position, the film thickness is equal to the minimum film thickness, $h_m$. At the edge position $x = \frac{L}{2}$, the film thickness is equal to the sum of the minimum film thickness and the amplitude,
\((h_m + d)\). The advantage of the proposed equation (4.2.1) is that, for a fixed length \(L\), one can generate different sizes of convex curved surfaces by varying the value of amplitudes for the cosine function. The study of combined effects of non-Newtonian rheology and pressure-dependent viscosity in the sphere-plate system is made by Lu and Lin (2007). In the present problem, it is assumed that, the lubricant in the film region is Eringen’s (1966) micropolar fluid.

The modified Reynolds equation for the squeeze film lubrication of bearings lubricated by the micropolar fluids was derived by Allen et. al. (1971) in the following form

\[
\frac{\partial}{\partial \chi} \left[ \frac{f(N, l, h)}{\mu} \frac{\partial p}{\partial \chi} \right] + \frac{\partial}{\partial \zeta} \left[ \frac{f(N, l, h)}{\mu} \frac{\partial p}{\partial \zeta} \right] = 12 \frac{\partial h}{\partial t}
\]  

(4.2.2)

where,

\[
f(N, l, h) = h^3 + 12l^2h - 6Nlh^2 \coth\left(\frac{Nh}{2l}\right)
\]

\[
\frac{\partial h}{\partial t} = c \frac{\partial \varepsilon}{\partial t} \cos \theta;
\]

\[
N = \left(\frac{\chi}{\chi + 2\mu}\right)^\frac{1}{3}, \quad m = \frac{N}{l}, \quad l = \left(\frac{\gamma}{4\mu}\right)^\frac{1}{2};
\]

For the axisymmetric case equation (4.2.2) reduce to

\[
\frac{\partial}{\partial \chi} \left[ \frac{f(N, l, h)}{\mu} \frac{\partial p}{\partial \chi} \right] = 12h_m'
\]  

(4.2.3)

In this equation, \(h_m'\) denotes \(\frac{dh_m}{dt}\), where \(t\) is the time. In order to analyze the problem conveniently, the following non-dimensional variables and parameters are introduced.

\[
x^* = \frac{x}{L}, \quad h_m^* = \frac{h_m}{h_m^0}, \quad h^* = \frac{h}{h_m^0}, \quad p^* = \frac{ph_m^3}{\mu_0 L^2 (-h_m^*)}, l^* = \frac{l}{h_m^0}, \quad \mu = \mu_0 e^{\mu p}
\]  

(4.2.4)
into equation (4.2.3) to yield the non-dimensional modified Reynolds equation and the non-dimensional film thickness in the form

Fig.4.1 Squeeze film geometry between convex curved plates with cosine form
\[
\frac{d}{dx} \left[ \left( f^*(N, I', h^*) \right) e^{-p^*} \right] \frac{dp^*}{dx} = -12 \tag{4.2.5}
\]

Where,

\[
h^* = h_m^* + A[1 - \cos(\pi x^*)], \quad A = \frac{d}{h_{m0}}, \quad P = \frac{\mu_0 a L^2(-h_m^*)}{h_{m0}}, \tag{4.2.6}
\]

In this equation, \(A\) is the amplitude ratio of the cosine-form convex curved plates and \(P\) is the PDV parameter, respectively.

The pressure conditions are:

\[
p^* = 0 \quad \text{at} \quad x^* = \pm \frac{1}{2} \quad \text{and} \quad \frac{dp^*}{dx^*} = 0 \quad \text{at} \quad x^* = 0. \tag{4.2.7}
\]

Integration of equation (4.2.5) with respect to \(x^*\) and the use of boundary condition (4.2.7) yield the expression for the fluid film pressure in the form

\[
p^* = -\frac{1}{P} \ln \left\{ 1 - \frac{1}{2} \int_{x^*}^{\frac{1}{2}} \left[ f^*(N, I', h^*) \right] dx^* \right\} \tag{4.2.8}
\]

Integrating the film pressure, one can obtain the load-carrying capacity

\[
W = \int_{x^* = -\frac{L}{2}}^{\frac{L}{2}} \ p.D\ dx \tag{4.2.9}
\]

where \(D\) denotes the width of the curved plates. The non dimension load carrying capacity is obtained in the form

\[
W^* = \frac{Wh_m^3}{\mu_0 L^2 D(-h_m^*)} = \int_{x^* = -\frac{1}{2}}^{\frac{1}{2}} p^*\ dx^* \tag{4.2.10}
\]

Use of equation (4.2.8) in (4.2.10) gives the non-dimensional load capacity in the form
\[ W^* = -\frac{1}{P} \int_{x^* = \frac{1}{2}}^{\frac{1}{2}} \ln [1 - \int_{x^* = \frac{1}{2}}^{\frac{1}{2}} \frac{12P x^*}{f^*(N, l^*, h^*)} dx^*] dx^* \]  

(4.2.11)

Introducing the non-dimensional time as

\[ t^* = \frac{Wh2}{\mu_0 L D(-h''_m)} \]  

(4.2.12)

Where, \( h''_m = \frac{dh^*_m}{dt} \)  

(4.2.13)

The use of (4.2.13) in equation (4.2.12) yields

\[ \frac{dh^*_m}{dt} = P \int_{x^* = \frac{1}{2}}^{\frac{1}{2}} \ln [1 - \int_{x^* = \frac{1}{2}}^{\frac{1}{2}} \frac{12P x^*}{f^*(N, l^*, h^*)} dx^*] dx^* \]  

(4.2.14)

The Initial condition for the minimum film thickness is: \( h^*_m(t^* = 0) = 1 \). After integrating the differential equation, the elapsed time required for the upper curved plate to approach the lower plate can be derived in the form

\[ t^* = -\frac{1}{P} \int_{h''_m x^* = \frac{1}{2}}^{\frac{1}{2}} \ln [1 - \int_{x^* = \frac{1}{2}}^{\frac{1}{2}} \frac{12P x^*}{f^*(N, l^*, h^*)} dx^*] dx^* dh^*_m \]  

(4.2.15)

Using the numerical method of integration, the film pressure (4.2.8), the load capacity (4.2.11) and the elapsed time (4.2.15) can be calculated. In the limiting case of \( l \rightarrow 0, N \rightarrow 0 \), equations (4.2.8), (4.2.11) and (4.2.15) reduce to the corresponding Newtonian case studied by Lin et. al.

4.3 Results and discussions

The effects of pressure dependent viscosity on the squeeze film characteristics of micropolar fluid in convex curved plates is analyzed. The results are analyzed with
respect to various non-dimensional parameters such as the coupling number, $N$, the additives length size parameter, $l'$ the amplitude ratio $A$. The following range of parameters are used for the discussion of the convex curved plate.

$N=0, 0.2, 0.4$;

$l'=0, 0.2, & 0.4$;

$A=0, 0.1 & 0.2$;

$P=0, 0.002 & 0.004$;

4.3.1 Squeeze film pressure

The variation of non-dimensional squeeze film pressure $p'$ as a function of horizontal co-ordinate $x'$ for different values of $N$ is depicted in figure 4.2. It is observed that, $p'$ increases for the increasing values of $N$. Further, it is observed that $p'$ also increases for the micropolar lubricant as compared to the corresponding Newtonian case.

Figure 4.3 shows the film pressure $p'$ as a function of the horizontal coordinate $x'$ for different values of the amplitude ratio $A$ and the PDV parameter $P$. From the definition of the geometry in equation (4.2.6), the cosine form convex curved plates reduce to the parallel flat plates as the value of the amplitude ratio $A \rightarrow 0$. Observing the displayed results, the peak values of the film pressure for the cosine form convex curved plates are lower than those of the parallel flat plates. Under the amplitude ratio, the PDV effects $P=0.002$ results in a higher film pressure as compared to the iso-viscous case. Increasing the value of the PDV parameter ($P=0.004$) increases the film pressure. When the amplitude ratio decreases down or the situation of flat plates, the increased amounts of the film pressure due to the effects of PDV are more emphasized.
4.3.2 Load carrying capacity

The variation of non-dimensional load carrying capacity $W^*$ as a function of the amplitude ratio $A$ for different values of the minimum film thickness $h_m^*$ and the PDV parameter $P$. Figure 4.4 shows that for the various values of $N$, the value of $W^*$ is seen to decrease with increasing $A$. Figure 4.5 shows that at the film thickness $h_m^* = 0.3$, the value of $W^*$ is seen to decrease with increasing $A$. In iso-viscous case, the increments of the load capacity the effect of PDV are observed. It is observed that decreasing the value of $h_m^*$ increases the influences of $P$ on the load capacity of the convex curved plates. In figure 5.5 we observed that $W^*$ decreases for the increasing value $A$ for different microstructure size parameter $l'$, for fixed values of $N=0.2$, $P=0.002$, $h_m^* = 0.2$.

4.3.3 Squeeze film time

Figure 4.6 presents the minimum film height $h_m^*$ as a function of the elapsed time $t^*$ for different values of $N$. The film thickness decreases for increasing the value of non-dimension $t^*$ with $l' = 0.4$, $P = 0.002$, $A = 0.1$. In figure 4.6 we observed that a longer elapsed time for the convex plates. Increasing the PDV parameter increases the longer time for the upper curved plate required to achieve a prescribed film height. When time increases the film thickness decreases. Figure 4.7 shows that the variation of non-dimensional film thickness $h_m^*$ as a function of amplitude ratio $A$ for different value of $l'$. When the value of $t^*$ increases the $h_m^*$ decreases for fixed values of $P = 0.004$, $N = 0.6$, $A = 0.1$.

On the whole, the elapsed times for the cosine-form convex curved plates are lengthened by considering the effects of PDV of lubricants.
4.4 Conclusions

The effect of pressure dependent viscosity on the micropolar fluid lubricated squeeze film in convex curved plates is analyzed. It is found that, the effect of pressure dependent viscosity is to increase the load carrying capacity and the squeeze film time for cosine form curved plates as compared to the corresponding iso-viscous case. These effects are more accentuated for larger value of PDV parameter and a smaller amplitude ratio.
Fig. 4.2 Variation of non-dimensional $p^*$ with $x^*$ for different values of $N$ with $p=0.002$, $h_m=0.3$, $A=0.1$. 
Fig. 4.3 Variation of non-dimensional $p^*$ with $x^*$ for different value of $A$ with $l^* = 0.4, N = 0.2, h_m = 0.2$. 
Fig. 4.4 Variation of non-dimensional load $w^*$ with $A$ for different value of $N$ with $P=0.002$, $h_m=0.3$. 

- $i^* = 0.4$
- $i^* = 0.0$
- $N=0.0$ (solid line)
- $N=0.2$ (dotted line)
- $N=0.4$ (dashed line)
Fig. 4.5 Variation of non-dimensional load $w^*$ with $A$ for different values of $P$ with $l^* = 0.4$, $N = 0.2$, $h_{m^*} = 0.2$. 

$h_{m^*} = 0.2$  \hspace{1cm}  $h_{m^*} = 0.3$  
- $P = 0.0$  \hspace{1cm}  $P = 0.0$  
- $P = 0.002$  \hspace{1cm}  $P = 0.002$  
- $P = 0.004$  \hspace{1cm}  $P = 0.004$
Fig. 4.5. Variation of non-dimensional load $w^*$ with $A$ for different value of $P$ with $l^*=0.4$, $N=0.2$, $h_m=0.2$. 
$h_m=0.2$ $h_m=0.3$
$P=0.0$ $P=0.0$
$P=0.002$ $P=0.002$
$P=0.004$ $P=0.004$
Fig. 4.6. Variation of non-dimensional time $t^*$ with minimum film height $h_m^*$ for different values of $N$ with $P=0.002$, $A=0.2$. 

- $l^*=0.4$ 
- $l^*=0.2$ 
- $N=0.0$ 
- $N=0.2$ 
- $N=0.4$
Fig 4.7. Variation of non-dimensional time $t^*$ with minimum film height $h_m^*$ for different values of $P$ with $l^* = 0.4$, $N = 0.2$, $A = 0.1$. 

$A = 0.0$  
$A = 0.1$  
$P = 0.0$  
$P = 0.002$  
$P = 0.004$