CHAPTER V

ON THE EXPECTED PROFIT OF A SYSTEM
WITH REST PERIOD, PATIENCE TIME
AND VARIOUS TYPES OF REPAIR
ON THE EXPECTED PROFIT OF A SYSTEM WITH REST PERIOD, PATIENCE TIME AND VARIOUS TYPES OF REPAIR

Introduction

In Chapter III and IV, we examine the systems with one of the assumptions that the expert comes to the system if regular/assistant repairman finds himself unable to complete the repair. However, there may be situations when we cannot wait for a long time when regular/assistant repairman is trying to repair the failed unit i.e. one can wait only up to some tolerable (patience) time.

In this chapter, we introduce the concept of patience time together with three types of repairmen in a two-unit cold standby system. One of the repairmen is regular and the others are visiting (an expert and his assistants). The regular/assistant repairman may not be able to do some complex repairs. However, the expert repairman does the repairs perfectly. If the regular repairman gets tired, an assistant of the expert comes to resume the repair. The expert comes immediately to take over the system in any of the following occurrences:

(i) The regular/assistant repairman is unable to repair the unit within the patience time.

(ii) The system becomes inoperative

However, if the assistant/regular repairman finds himself unable to complete the repair, the expert takes his own time to arrive at the system. Another model is also discussed wherein there is no provision
of calling the assistant repairman i.e. we wait till the completion of rest period of the regular repairman or till the system becomes inoperable. Other assumptions are as usual.

Various measures of system effectiveness such as mean time to system failure, steady state availability, busy period analysis of the assistant/expert repairman, expected number of visits by the assistant/expert repairman and expected profit earned by the system are determined. Graphs are plotted for a particular case. Profits of the two models are also compared graphically.

Notations

$\lambda$ \hspace{1cm} constant failure rate

$a$ \hspace{1cm} the probability that the regular/assistant repairman is able to repair the failed unit

$b$ \hspace{1cm} the probability that the regular repairman does not need rest

$p = ab$

$q = a(1-b)$

$p_l$ \hspace{1cm} the probability that the regular repairman is available on the completion of repair by the assistant/expert repairman

$q_l$ \hspace{1cm} the probability that the regular repairman is not available

$g(t),G(t)$ \hspace{1cm} p.d.f. and c.d.f. of repair time of the regular repairman

$g_a(t),G_a(t)$ \hspace{1cm} p.d.f. and c.d.f. of repair time of the assistant repairman

$g_e(t),G_e(t)$ \hspace{1cm} p.d.f. and c.d.f. of repair time of the expert repairman

$i(t),I(t)$ \hspace{1cm} p.d.f. and c.d.f. of patience time

$w(t),W(t)$ \hspace{1cm} p.d.f. and c.d.f. of the waiting time for repair to be done by the expert repairman
h_(t),H_(t) \quad \text{p.d.f. and c.d.f. of the waiting time for the repair when the regular repairman is taking rest}

E_1(t) = g(t) \tilde{I}(t)
E_2(t) = i(t) \tilde{G}(t)
E_3(t) = \tilde{G}(t) \tilde{I}(t)
F_1(t) = g_a(t) \tilde{I}(t)
F_2(t) = i(t) \tilde{G}_a(t)
F_3(t) = \tilde{G}_a(t) \tilde{I}(t)

Symbols for the states of the system are:

- o: operative unit
- cs: cold standby
- o_n: operative unit (suffix n represents that the regular repairman is not available)
- Fr: failed unit under repair of the regular repairman
- Fra: failed unit under repair of the assistant repairman
- Fre: failed unit under repair of the expert repairman
- FRe: repair of the failed unit is continuing by the expert repairman from the previous state
- F_w: failed unit waiting for repair to be done by expert
- F_{wr}: failed unit waiting for repair when regular repairman is taking rest

Model 1

The state transition diagram for the model is shown as in Fig. 5.1. In this model, there is provision of all types of repairman i.e. regular, expert and his assistants.
Fig. 5.1.
Transition Probabilities and Mean Sojourn Times

The epochs of entries into states 0, 1, 2, 3, 4, 5 and 6 are regeneration points and thus states 0, 1, 2, 3, 4, 5 and 6 are regenerative states. States 5 and 7 are down states. The transition probabilities are:

\[ q_{01}(t) = \lambda e^{-\lambda t}; \quad q_{10}(t) = p e^{-\lambda t} \bar{g}(t) \quad \bar{I}(t) = p e^{-\lambda t} E_1(t) \]

\[ q_{12}(t) = q e^{-\lambda t} \bar{g}(t) \quad \bar{I}(t) = q e^{-\lambda t} E_1(t) \]

\[ q_{13}(t) = e^{-\lambda t} i(t) \quad \bar{G}(t) = e^{-\lambda t} E_2(t) \]

\[ q_{14}(t) = (1-a) e^{-\lambda t} \bar{g}(t) \quad \bar{I}(t) = (1-a) e^{-\lambda t} E_1(t) \]

\[ q_{15}(t) = \lambda e^{-\lambda t} \quad \bar{G}(t) = \lambda e^{-\lambda t} E_3(t) \]

\[ q_{20}(t) = p_1 ae^{-\lambda t} \bar{g}(t) \quad \bar{I}(t) = p_1 a e^{-\lambda t} F_1(t) \]

\[ q_{23}(t) = e^{-\lambda t} i(t) \quad \bar{G}_a(t) = e^{-\lambda t} F_2(t) \]

\[ q_{24}(t) = (1-a) e^{-\lambda t} \bar{g}(t) \quad \bar{I}(t) = (1-a) e^{-\lambda t} F_1(t) \]

\[ q_{25}(t) = \lambda e^{-\lambda t} \quad \bar{G}_a(t) = \lambda e^{-\lambda t} F_3(t) \]

\[ q_{26}(t) = q_1 ae^{-\lambda t} \bar{g}(t) \quad \bar{I}(t) = q_1 a e^{-\lambda t} F_1(t) \]

\[ q_{30}(t) = p_1 e^{-\lambda t} \bar{g}(t); \quad q_{36}(t) = q_1 e^{-\lambda t} \bar{g}(t) \]

\[ q_{37}(t) = \lambda e^{-\lambda t} \bar{G}_e(t) \]

\[ q_{33}^{(7)}(t) = [\lambda e^{-\lambda t} \otimes 1] \bar{g}_e(t) = [1-e^{-\lambda t}] \bar{g}_e(t) \]

\[ q_{43}(t) = e^{-\lambda t} w(t) \]

\[ q_{44}(t) = \lambda e^{-\lambda t} \bar{W}(t) \]

\[ q_{53}(t) = \bar{g}_e(t); \quad q_{61}(t) = p_1 \lambda e^{-\lambda t} \]

\[ p_{62}(t) = q_1 \lambda e^{-\lambda t} \quad (5.1.1 - 5.1.20) \]

The non-zero elements \( p_{ij} \) are given by

\[ p_{01} = 1, \quad p_{10} = p E_1^*(\lambda), \quad p_{12} = q E_1^*(\lambda), \quad p_{13} = E^*(\lambda) \]

\[ p_{14} = (1-a)E_1^*(\lambda), \quad p_{15} = \lambda E_3^*(\lambda), \quad p_{20} = p_1 a F_1^*(\lambda), \quad p_{23} = F_2^*(\lambda) \]
\( p_{24} = (1-a)F_1^*(\lambda), \, p_{25} = \lambda F_3^*(\lambda), \, p_{26} = q_1 a F_1^*(\lambda), \, p_{30} = p_1 g_e^*(\lambda) \)
\( p_{36} = q_1 g_e^*(\lambda), \, p_{37} = 1 - g_e^*(\lambda), \, p_{33}^{(7)} = 1 - g_e^*(\lambda), \, p_{43} = w^*(\lambda) \)
\( p_{45} = 1 - w^*(\lambda), \, p_{53} = 1, \, p_{61} = p_1, \, p_{62} = q_1 \) (5.1.21 - 5.1.40)

By these transition probabilities, it can be verified that
\( p_{01} = 1, \, p_{10} + p_{12} + p_{13} + p_{14} + p_{15} = 1, \, p_{20} + p_{23} + p_{24} + p_{25} + p_{26} = 1, \)
\( p_{30} + p_{36} + p_{37} = 1, \, p_{30} + p_{36} + p_{33}^{(7)} = 1, \, p_{43} + p_{45} = 1 \)
\( p_{61} + p_{62} = 1, \, p_{53} = 1 \) (5.1.41 - 5.1.48)

The mean sojourn times \( \mu_i \) are:
\[
\begin{align*}
\mu_0 &= \frac{1}{\lambda}, \quad \mu_1 = E_3^*(\lambda), \quad \mu_2 = F_3^*(\lambda), \quad \mu_3 = \frac{1 - g_e^*(\lambda)}{\lambda} \\
\mu_4 &= \frac{1 - w^*(\lambda)}{\lambda}, \quad \mu_5 = \int_0^\infty \lambda G_e(t) dt = \int_0^\infty \lambda g_e(t) dt, \quad \mu_6 = \frac{1}{\lambda} = \mu_0
\end{align*}
\] (5.1.49 - 5.1.55)

The unconditional mean time taken by the system to transit for any state \( j \) when it is counted from epoch of entrance into state \( i \) is mathematically stated as:
\[
m_{ij} = \int_0^\infty q_{ij}(t) dt = -q_{ij}^*(0) \] (5.1.56)

Thus,
\[
\begin{align*}
m_{01} &= \mu_0, \quad m_{10} + m_{12} + m_{13} + m_{14} + m_{15} = \mu_1 \\
m_{20} + m_{23} + m_{24} + m_{25} + m_{26} &= \mu_2 \\
m_{30} + m_{36} + m_{37} &= \mu_3, \quad m_{30} + m_{36} + m_{33}^{(7)} = \mu_5 \\
m_{43} + m_{45} &= \mu_4; \quad m_{53} = \mu_5 \\
m_{61} + m_{62} &= \mu_6
\end{align*}
\] (5.1.57 - 5.1.64)
Mean Time to System Failure

By probabilistic arguments, we obtain the following recursive relations for \( p_i(t) \):

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t)\phi_1(t) \\
\phi_1(t) &= Q_{10}(t)\phi_0(t) + Q_{12}(t)\phi_2(t) + Q_{13}(t)\phi_3(t) \\
&\quad + Q_{14}(t)\phi_4(t) + Q_{15}(t) \\
\phi_2(t) &= Q_{20}(t)\phi_0(t) + Q_{23}(t)\phi_3(t) + Q_{24}(t)\phi_4(t) + Q_{25}(t) \\
&\quad + Q_{26}(t)\phi_6(t) \\
\phi_3(t) &= Q_{30}(t)\phi_0(t) + Q_{36}(t)\phi_6(t) + Q_{37}(t) \\
\phi_4(t) &= Q_{43}(t)\phi_3(t) + Q_{45}(t) \\
\phi_5(t) &= Q_{50}(t)\phi_1(t) + Q_{52}(t)\phi_2(t)
\end{align*}
\]

(5.1.65 - 1.5.70)

Taking L.S.T. of these relations and solving them for \( \phi_0^{**}(s) \), we have

\[
\phi_0^{**}(s) = \frac{N(s)}{D(s)}
\]

(5.1.71)

where

\[
N(s) = Q_{01}^{**}(s)[(Q_{12}^{**}(s)Q_{24}^{**}(s) - Q_{62}^{**}(s)Q_{26}^{**}(s)Q_{14}^{**}(s)) \\
(Q_{37}^{**}(s)Q_{43}^{**}(s) + Q_{45}^{**}(s)) + Q_{62}^{**}(s)](Q_{36}^{**}(s)Q_{13}^{**}(s)) \\
(Q_{25}^{**}(s) + Q_{24}^{**}(s)Q_{45}^{**}(s)) + Q_{36}^{**}(s)Q_{14}^{**}(s)(Q_{25}^{**}(s)) \\
Q_{43}^{**}(s) - Q_{45}^{**}(s)Q_{23}^{**}(s) - Q_{26}^{**}(s)(Q_{15}^{**}(s) + Q_{13}^{**}(s)) \\
Q_{37}^{**}(s) - Q_{36}^{**}(s)Q_{15}^{**}(s)Q_{23}^{**}(s) + Q_{43}^{**}(s)Q_{24}^{**}(s)) \\
+ Q_{12}^{**}(s)Q_{37}^{**}(s)Q_{23}^{**}(s) + Q_{25}^{**}(s)Q_{12}^{**}(s) + Q_{37}^{**}(s) \\
(Q_{13}^{**}(s) + Q_{14}^{**}(s)Q_{43}^{**}(s)) + (Q_{15}^{**}(s) + Q_{14}^{**}(s)Q_{45}^{**}(s))] \\
D(s) = 1 - (Q_{43}^{**}(s)Q_{24}^{**}(s)Q_{36}^{**}(s) + Q_{23}^{**}(s)Q_{36}^{**}(s) + Q_{26}^{**}(s)) \\
(Q_{62}^{**}(s) + Q_{61}^{**}(s)Q_{12}^{**}(s) - Q_{01}^{**}(s)Q_{62}^{**}(s)Q_{10}^{**}(s)) \\
- (Q_{43}^{**}(s)Q_{14}^{**}(s) + Q_{13}^{**}(s))Q_{61}^{**}(s)Q_{36}^{**}(s) \\
+ Q_{01}^{**}(s)Q_{62}^{**}(s)(Q_{20}^{**}(s)Q_{36}^{**}(s) - Q_{26}^{**}(s)Q_{30}^{**}(s))]
\]
Now, the mean time to system failure (MTSF) when the system starts from the state '0' is

\[ T_0 = \lim_{s \to 0} \frac{1 - \phi_0**(s)}{s} = \frac{N}{D} \quad (5.1.74) \]

where

\[ N = p_{36}(p_{23}+p_{24}p_{43}) \left[ p_{12}u_3-p_{62}\{(1-p_{12})\mu_0 + \mu_1\}\right] \]

\[ + (p_{13}+p_{14}p_{43})[p_{36}(\mu_0 + p_{62}\mu_2) + (1-p_{26}p_{62})\mu_3] \]

\[ + (1-p_{26}p_{62})(\mu_0+ \mu_1 + p_{14}\mu_4) + p_{12}[p_{26}p_{62}\mu_0+\mu_1+p_{24}\mu_4] \]

\[ + p_{36}p_{62}(p_{24}p_{13} - p_{14}p_{23})\mu_4 \]

\[ D = 1-p_{10}+p_{12}p_{20} - \{1-p_{61}(1-p_{12}) - p_{62}p_{10}\}[p_{26}+p_{36}(p_{23}+p_{24}p_{43})] \]

\[ -(p_{13}+p_{14}p_{43})[p_{36}p_{61}+p_{62}(p_{20}p_{36} - p_{30}p_{26})] - p_{30}\{(p_{13}+p_{12}p_{23}) \]

\[ + p_{43}(p_{14}+p_{12}p_{24})\} \quad (5.1.75 - 5.1.76) \]

### Availability Analysis

The recursive relations for \( A_i(t) \) are as follows:

\[ A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) \]

\[ A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t) + q_{12}(t) \odot A_2(t) + q_{13}(t) \odot A_3(t) \]

\[ + q_{14}(t) \odot A_4(t) + q_{15}(t) \odot A_5(t) \]

\[ A_2(t) = M_2(t) + q_{20}(t) \odot A_0(t) + q_{23}(t) \odot A_3(t) + q_{24}(t) \odot A_4(t) \]

\[ + q_{25}(t) \odot A_5(t) + q_{26}(t) \odot A_6(t) \]

\[ A_3(t) = M_3(t) + q_{30}(t) \odot A_0(t) + q_{36}(t) \odot A_6(t) + q_{33}^{(7)}(t) \odot A_3(t) \]

\[ A_4(t) = M_4(t) + q_{43}(t) \odot A_3(t) + q_{45}(t) \odot A_5(t) \]

\[ A_5(t) = q_{53}(t) \odot A_3(t) \]

\[ A_6(t) = M_6(t) + q_{61}(t) \odot A_1(t) + q_{62}(t) \odot A_2(t) \quad (5.1.77 - 5.1.83) \]
where

\[ M_0(t) = e^{-\lambda t}, \quad M_1(t) = e^{-\lambda t} E_3(t), \quad M_2(t) = e^{-\lambda t} F_3(t) \]
\[ M_3(t) = e^{-\lambda t} \bar{G}_c(t), \quad M_4(t) = e^{-\lambda t} \bar{W}(t), \quad M_6(t) = e^{-\lambda t} \]

(5.1.84–5.1.89)

Taking the Laplace transforms of the above equations and solving them for \( A_0^*(s) \), we get

\[ A_0^*(s) = \frac{N_1(s)}{D_1(s)} \]  

(5.1:90)

where

\[ N_1(s) = \begin{cases} 1-q_{33}^{(7)}(s) & \{ M_0^*(s)(1-q_{62}^*(s)q_{26}^*(s)-q_{61}^*(s)q_{12}^*(s)q_{26}^*(s)) \\
+ q_{01}^*(s)q_{12}^*(s)M_6^*(s)q_{26}^*(s) & + q_{01}^*(s)q_{12}^*(s)(q_{24}^*(s)M_4^*(s) \\
+ M_2^*(s)) & + (1-q_{62}^*(s)q_{26}^*(s))(M_1^*(s)+M_4^*(s)q_{14}^*(s)) \end{cases} \]

\[ + q_{33}^*(s)(q_{24}^*(s)q_{45}^*(s) & + q_{25}^*(s))[-M_0^*(s)q_{36}^*(s)(q_{62}^*(s) \\
+ q_{61}^*(s)q_{42}^*(s)) & + q_{01}^*(s)\{q_{12}^*(s)(M_3^*(s)+q_{36}^*(s)M_6^*(s)) \\
- M_1q_{36}^*(s)q_{62}^*(s) & \} - (q_{14}^*(s)q_{45}^*(s) & + q_{15}^*(s))q_{53}^*(s) \]

\[ \{M_0^*(s)q_{61}^*(s)q_{36}^*(s) & + q_{61}^*(s)(M_3^*(s)q_{62}^*(s)q_{26}^*(s) \\
- M_3^*(s)-q_{36}^*(s)M_6^*(s) & - q_{36}^*(s)q_{62}^*(s)M_2^*(s) \} \} \]

\[ + \{ q_{13}^*(s)+q_{14}^*(s)q_{45}^*(s) \} \{ q_{36}^*(s)(M_2^*(s)q_{62}^*(s)q_{01}^*(s) \\
- M_0^*(s)q_{61}^*(s)) & + q_{01}^*(s)\{M_6^*(s)q_{36}^*(s) \\
+ M_3^*(s)(1-q_{62}^*(s)q_{26}^*(s)) & \} - (q_{23}^*(s)+q_{24}^*(s)q_{45}^*(s) \]

\[ [q_{36}^*(s)(M_0q_{62}^*(s)+M_0^*(s)q_{61}^*(s)q_{12}^*(s) \\
- q_{01}^*(s)q_{12}^*(s)M_6^*(s) & - q_{01}^*(s)q_{62}^*(s)M_1^*(s)) \]

\[ + q_{01}^*(s)q_{12}^*(s)M_3^*(s) & ] + M_4^*(s)q_{36}^*(s)q_{62}^*(s) \]

\[ [\{q_{13}^*(s)q_{24}^*(s)-q_{14}^*(s)q_{23}^*(s) & - q_{01}^*(s)q_{33}^*(s)(q_{14}^*(s)q_{25}^*(s) \\
- q_{15}^*(s)q_{24}^*(s)) \} \]

\[ D_1(s) = (1-q_{33}^{(7)}(s))[1-q_{26}^*(s)q_{62}^*(s)+q_{61}^*(s)q_{12}^*(s)) - q_{01}^*(s)q_{10}^*(s) \]

\[ (1-q_{62}^*(s)q_{26}^*(s))-q_{01}^*(s)q_{12}^*(s)q_{20}^*(s)]-[\{q_{23}^*(s) \\
+ q_{24}^*(s)q_{43}^*(s)) & + q_{53}^*(s)(q_{25}^*(s)+q_{24}^*(s)q_{45}^*(s)) \} \]
\[\begin{align*}
&[q_{36}(s)(q_{62}(s)+ q_{61}(s)q_{12}(s))+q_{30}(s)q_{01}(s)q_{12}(s) \\
&-q_{01}(s)q_{10}(s)q_{36}(s)q_{62}(s)] - [(q_{13}(s)+q_{14}(s)q_{4}(s)) \\
&+q_{53}(s)(q_{14}(s)q_{45}(s)+q_{15}(s))] - [q_{36}(s)q_{61}(s)+q_{30}(s)q_{01}(s) \\
&+ q_{01}(s)q_{62}(s)(q_{20}(s)q_{36}(s)-q_{30}(s)q_{26}(s))] \\
&\text{(5.1.91–5.1.92)}
\end{align*}\]

In steady state the availability of the system is given by

\[A_0 = N_1/D_1\]  \hspace{1cm} (5.1.93)

where

\[N_1 = (p_{30}+p_{36})[(1-p_{26}p_{62})(\mu_0+\mu_1+p_{14}\mu_4) + (p_{12}p_{26}p_{62}\mu_0) \\
+p_{12}\mu_2+p_{24}\mu_4)]+(p_{23}+p_{24}+p_{25})[p_{12}(\mu_3+p_{36}p_{62}\mu_0) \\
-p_{36}p_{62}(\mu_1+\mu_4)]+(p_{13}+p_{14}+p_{15})[p_{36}p_{62}(\mu_0+\mu_2) \\
+(1-p_{26}p_{62})\mu_3]+[p_{36}p_{62}(p_{13}p_{24}-p_{14}p_{23})+(p_{15}p_{24}) \\
-p_{14}p_{23})]\mu_4\]

\[D_1 = [(p_{30}+p_{36})\{p_{12}(p_{20}+p_{26}) +p_{10}(1-p_{26}p_{62})\}+(p_{23}+p_{24}+p_{25}) \\
[p_{12}(p_{30}+p_{36})+p_{10}p_{36}p_{62})+(p_{13}+p_{14}+p_{15})\{(p_{30}+p_{36}) \\
+p_{62}(p_{20}p_{36}-p_{30}p_{26}))]\mu_0+[p_{36}p_{61}+p_{30}+p_{62}(p_{20}p_{36} \\
-p_{30}p_{26})]\mu_1+p_{14}\mu_4+(p_{15}p_{14}\mu_4)\mu_5]+[p_{36}(p_{62}+p_{61}p_{12} \\
+p_{30}p_{12}-p_{10}p_{36}p_{62})[p_{24}\mu_4+(p_{25}+p_{24}p_{45})\mu_5] \\
+[p_{36}p_{62}(p_{13}+p_{14}+p_{15})+p_{12}(p_{30}+p_{36})]\mu_2 \\
+p_{61}p_{12}+p_{30}p_{12}-p_{10}p_{36}p_{62})[p_{24}\mu_4+(p_{15}+p_{44}p_{45})\mu_5] \\
+[p_{36}(p_{62}+p_{61}p_{12})+p_{30}p_{12}-p_{10}p_{36}p_{62})[p_{24}\mu_4+(p_{25} \\
+p_{24}p_{45})\mu_5]+[p_{36}p_{62}(p_{13}+p_{14}+p_{15})+p_{12}(p_{30}+p_{36})] \mu_2 \\
+[p_{12}(p_{23}+p_{24}+p_{25})+(p_{13}+p_{14}+p_{15})(1-p_{26}p_{62})] \mu_5 \\
\text{(5.1.94–5.1.95)}\]
Busy Period Analysis of the Expert Repairman

By probabilistic arguments, we have the following recursive relations:-

\[ B_0^e(t) = q_{01}(t) \odot B_1^e(t) \]

\[ B_1^e(t) = q_{10}(t) \odot B_0^e(t) + q_{12}(t) \odot B_2^e(t) + q_{13}(t) \odot B_3^e(t) + q_{14}(t) \odot B_4^e(t) + q_{15}(t) \odot B_5^e(t) \]

\[ B_2^e(t) = q_{20}(t) \odot B_0^e(t) + q_{23}(t) \odot B_3^e(t) + q_{24}(t) \odot B_4^e(t) + q_{25}(t) \odot B_5^e(t) + q_{26}(t) \odot B_6^e(t) \]

\[ B_3^e(t) = w_3(t) + q_{30}(t) \odot B_0^e(t) + q_{36}(t) \odot B_6^e(t) + q_{33}(t) \odot B_3^e(t) \]

\[ B_4^e(t) = q_{43}(t) \odot B_3^e(t) + q_{45}(t) \odot B_5^e(t) \]

\[ B_5^e(t) = w_5(t) + q_{53}(t) \odot B_3^e(t) \]

\[ B_6^e(t) = q_{61}(t) \odot B_1^e(t) + q_{62}(t) \odot B_2^e(t) \]

(5.1.96-5.1.102)

where

\[ w_3(t) = G_3(t) = w_5(t) \]

(5.1.103)

Taking L.T. of the above equations and solving them for \( B_0^e^*(s) \), we get

\[ B_0^e^*(s) = N_2(s)/D_1(s) \]

(5.1.104)

where

\[ N_2(s) = q_{01}(s)w_3^*(s)[1-q_{33}(s)][(1-q_{26}(s)q_{62}(s))(q_{14}(s)q_{45}(s) + q_{15}(s))+q_{12}(s)(q_{24}(s)q_{45}(s) + q_{25}(s)) + (q_{43}(s) + q_{45}(s)q_{53}(s))\]

\[ \{ (1-q_{26}(s)q_{62}(s))q_{14}(s) + q_{12}(s)q_{24}(s)\} + (1-q_{26}(s)q_{62}(s)) \]

\[ (q_{13}(s) + q_{15}(s)q_{53}(s) + q_{12}(s)(q_{23}(s) + q_{25}(s)q_{53}(s)) + q_{62}(s)q_{36}(s)\} \]

\[ \{ q_{13}(s)(q_{25}(s) + q_{24}(s)q_{45}(s)) - q_{14}(s) \]

\[ (q_{23}(s)q_{45}(s) - q_{43}(s)q_{25}(s)) - q_{15}(s)q_{23}(s) \]

\[ + q_{34}(s)q_{43}(s)) \} \]

(5.1.105)
In steady state, the total fraction of time for which the system is under repair of the expert repairman is given by

$$B_0^c = N_2/D_1$$  \hspace{1cm} (5.1.106)

where

$$N_2 = \mu_5[(1-p_{26}p_{62})(p_{13}+p_{14}+p_{15}+(p_{30}+p_{36})(p_{15}+p_{14}p_{45})]+p_{12}(p_{23}+p_{24}+p_{25})$$

$$+(p_{25}+p_{24}p_{45})(p_{12}(p_{30}+p_{36})+p_{13}p_{36}p_{62})-p_{36}p_{62}(p_{23}(p_{15}+p_{14}p_{45})$$

$$+p_{43}(p_{15}p_{24} - p_{14}p_{25}))$$  \hspace{1cm} (5.1.107)

**Busy Period Analysis of Assistant Repairman**

By probabilistic arguments, we have the following recursive relations:

- \[B_0^a(t) = q_{01}(t)B_1^a(t)\]
- \[B_1^a(t) = q_{10}(t)B_0^a(t) + q_{12}(t)B_2^a(t) + q_{13}(t)B_3^a(t) + q_{14}(t)B_4^a(t) + q_{15}(t)B_5^a(t)\]
- \[B_2^a(t) = w_2(t) + q_{20}(t)B_0^a(t) + q_{23}(t)B_3^a(t) + q_{24}(t)B_4^a(t) + q_{25}(t)B_5^a(t)\]
- \[B_3^a(t) = q_{30}(t)B_0^a(t) + q_{36}(t)B_6^a(t) + q_{33}(t)B_3(t)\]
- \[B_4^a(t) = q_{43}(t)B_3^a(t) + q_{45}(t)B_5^a(t)\]
- \[B_5^a(t) = q_{53}(t)B_3^a(t)\]
- \[B_6^a(t) = q_{61}(t)B_1^a(t) + q_{62}(t)B_2^a(t)\]  \hspace{1cm} (5.1.108-5.1.114)

where

\[w_2(t) = e^{-\lambda t} \tilde{G}_a(t)\]  \hspace{1cm} (5.1.115)

Taking Laplace transform of above equations and solving them for \(B_0^a(s)\), we get

$$B_0^a(s) = N_3(s)/D_1(s)$$  \hspace{1cm} (5.1.116)

where

$$N_3(s) = q_{01}(s)w_2(s)[q_{12}(s)(1-q_{33}(s))^s] + q_{36}(s)q_{62}(s)$$
\[
\{q_{14}(s)(q_{43}(s)-q_{45}(s)q_{53}(s))+(q_{13}(s)+q_{15}(s)q_{53}(s))\}
\]

(5.1.117)

In steady state, the total fraction of time for which the system is under repair of the assistant repairman is given by

\[B_0 = N_3/D_1\]

(5.1.118)

where

\[N_3 = \mu_2[p_{12}(p_{30}+p_{36}) + p_{36}p_{62}(p_{13}+p_{14}+p_{15})]\]  

(5.1.119)

**Expected Number of Visits by the Expert Repairman**

The following recursive relations for \(V_i^e(t)\) are obtained:

\[V_0^e(t) = Q_{01}(t)S V_1^e(t)\]

\[V_1^e(t) = Q_{10}(t)S V_0^e(t) + Q_{12}(t)S V_2^e(t) + Q_{13}(t)S [1+V_3^e(t)] + Q_{14}(t)S V_4^e(t) + Q_{15}(t)S [1+V_5^e(t)]\]

\[V_2^e(t) = Q_{20}(t)S V_0^e(t) + Q_{23}(t)S [1+V_3^e(t)] + Q_{24}(t)S V_4^e(t) + Q_{25}(t)S V_5^e(t) + Q_{26}(t)S V_6^e(t)\]

\[V_3^e(t) = Q_{30}(t)S V_0^e(t) + Q_{36}(t)S V_6^e(t) + Q_{33}(t)S V_3^e(t)\]

\[V_4^e(t) = Q_{43}(t)S V_3^e(t) + Q_{45}(t)S V_5^e(t)\]

\[V_5^e(t) = Q_{53}(t)S V_3^e(t)\]

\[V_6^e(t) = Q_{61}(t)S V_1^e(t) + Q_{62}(t)S V_2^e(t)\]

(5.1.120 - 5.1.126)

Taking L.S.T. of the above equations and solving them for \(V_0^e(s)\), we get

\[V_0^e(s) = N_4(s)/D_1(s)\]

(5.1.127)

where

\[N_4(s) = q_{01}(s)(1-q_{33}(s))(1-q_{26}(s)q_{62}(s))\{(q_{13}(s)+q_{15}(s))\}
\]

\[+ q_{14}(s)(q_{43}(s)+q_{45}(s))\} + q_{12}(s)\{(q_{23}(s)+q_{25}(s))\}
\]

\[+ q_{24}(s)(q_{43}(s)+q_{45}(s))\} - q_{01}(s)q_{62}(s)q_{36}(s)q_{53}(s)\{(q_{13}(s)
\]

\[(q_{24}(s)q_{45}(s)+q_{25}(s))\} - q_{23}(s)(q_{14}(s)q_{45}(s)+q_{15}(s))\]
In steady state, the number of visits per unit time by the expert is given by

\[ V_0^e = \frac{N_4}{D_1} \quad (5.1.129) \]

where

\[ N_4 = (p_{30}+p_{36})[(1-p_{26}p_{62})(p_{13}+p_{14}+p_{15}) + p_{12}(q_{23}+p_{24}+p_{25})] \quad (5.1.130) \]

**Expected Number of Visits by the Assistant Repairman**

The following recursive relations for \( V_i^a(t) \) are obtained:

\[
V_0^a(t) = Q_{01}(t)S V_1^a(t)
\]

\[
V_1^a(t) = Q_{10}(t)S V_0^a(t) + Q_{12}(t)S[1+V_2^a(t)] + Q_{13}(t)S V_3^a(t) + Q_{14}(t)S V_4^a(t) + Q_{15}(t)S V_5^a(t)
\]

\[
V_2^a(t) = Q_{20}(t)S V_0^a(t) + Q_{23}(t)S V_3^a(t) + Q_{24}(t)S V_4^a(t) + Q_{25}(t)S V_5^a(t) + Q_{26}(t)S V_6^a(t)
\]

\[
V_3^a(t) = Q_{30}(t)S V_0^a(t) + Q_{36}(t)S V_6^a(t) + Q_{33}(7)S V_3^a(t)
\]

\[
V_4^a(t) = Q_{43}(t)S V_3^a(t) + Q_{45}(t)S V_5^a(t)
\]

\[
V_5^a(t) = Q_{53}(t)S V_3^a(t)
\]

\[
V_6^a(t) = Q_{61}(t)S V_1^a(t) + Q_{62}(t)S[1+V_2^a(t)] \quad (5.1.131-5.1.137)
\]

Taking L.S.T. of the above equations and solving them for \( V_0^{a**}(s) \), we get

\[ V_0^{a**}(s) = \frac{N_5(s)}{D_1(s)} \quad (5.1.138) \]

where

\[ N_5(s) = q_{01}*(s)q_{12}*(s)[(1-q_{33}*(7)^*(s))-(q_{23}*(s)+q_{25}*(s)q_{33}*(s))] \]
\[ \begin{aligned}
&+ q_{01}(s)q_{36}(s)q_{62}(s)[(q_{43}(s)+q_{45}(s)q_{53}(s))(q_{14}(s))

&- q_{12}(s)q_{24}(s)+q_{12}(s)q_{33}(s)(q_{25}(s)+q_{24}(s)q_{45}(s))

&+ (q_{23}(s)+q_{24}(s)q_{43}(s))+(q_{13}(s)+q_{15}(s)q_{53}(s)))]
\end{aligned} \]

(5.1.139)

In steady state, the expected number of visits per unit time by the assistant repairman is given by

\[ V_0^a = N_5/D_1 \] (5.1.140)

where

\[ N_5 = p_{12}(p_{30}+p_{36})+p_{36}p_{62}(p_{13}+p_{14}p_{15}) \] (5.1.141)

**Profit Analysis**

The expected total profit incurred to the system in steady state is given by

\[ P_1 = C_0A_0 - C_1B_0^c - C_2B_0^a - C_3V_0^c - C_4V_0^a - C_5 \] (5.1.142)

where

\[ C_0 = \text{revenue per unit up time of the system}. \]

\[ C_1 = \text{cost per unit time for which the expert repairman is busy} \]

\[ C_2 = \text{cost per unit time for which the assistant repairman is busy} \]

\[ C_3 = \text{cost per visit of the expert repairman} \]

\[ C_4 = \text{cost per visit of the assistant repairman} \]

\[ C_5 = \text{cost per unit time for the regular repairman} \]

**Particular Case**

Let us assume that \( g(t) = \alpha e^{-\alpha t}, \ g_\epsilon(t) = \alpha_1 e^{-\alpha_1 t}, \ g_\delta(t) = \alpha_2 e^{-\alpha_2 t}, i(t) = \gamma e^{-\gamma t}, \ w(t) = \beta e^{-\beta t}. \)

Therefore, we have

\[ p_{01} = 1, \ p_{10} = \frac{p\alpha}{\alpha + \gamma + \lambda}, \ p_{12} = \frac{q\alpha}{\alpha + \gamma + \lambda}. \]
Using the above equations and the equations (5.1.74), (5.1.93), (5.1.106), (5.1.118), (5.1.129), (5.1.140) and (5.1.142), we can obtain expressions for MTSF and profit for this particular case.

**Graphical Interpretation**

The above particular case is considered for the graphical representation.

Behaviour of MTSF and profit w.r.t. failure rate ($\lambda$) for different values of patience rate ($\gamma$) is shown as in Figs. 5.3 and 5.4 respectively. MTSF and profit both decrease with increase in the
values of failure rate. However, their values become higher for higher values of patience rate.

**Model 2**

The transition diagram showing various states of the system is shown as in Fig. 5.2. In this model, we consider that there is no provision of having any assistant repairman.

**Transition Probabilities and Mean Sojourn Times**

The epochs of entry into states 0, 1, 2, 3, 4, 5 and 6 are regeneration points and thus these states are regenerative states. States 5 and 7 are down states. Transition probabilities are:

\[
q_{01}(t) = \lambda e^{-\lambda t}
\]
\[
q_{10}(t) = p e^{-\lambda t} g(t) \quad \overline{I}(t) = p e^{-\lambda t} E_1(t)
\]
\[
q_{12}(t) = q e^{-\lambda t} g(t) \quad \overline{I}(t) = q e^{-\lambda t} E_1(t)
\]
\[
q_{13}(t) = e^{-\lambda t} i(t) \quad \overline{G}(t) = e^{-\lambda t} E_2(t)
\]
\[
q_{14}(t) = (1-a) e^{-\lambda t} g(t) \quad \overline{I}(t) = (1-a) e^{-\lambda t} E_1(t)
\]
\[
q_{15}(t) = \lambda e^{-\lambda t} \quad \overline{G}(t) \quad \overline{I}(t) = \lambda e^{-\lambda t} E_3(t)
\]
\[
q_{21}(t) = e^{-\lambda t} h_1(t) ;
\]
\[
q_{25}(t) = \lambda e^{-\lambda t} \quad \overline{H}_1(t)
\]
\[
q_{30}(t) = p_1 e^{-\lambda t} g_c(t);
\]
\[
q_{36}(t) = q_1 e^{-\lambda t} g_c(t)
\]
\[
q_{37}(t) = \lambda e^{-\lambda t} \quad \overline{G}_c(t)
\]
\[
q_{33}(7)(t) = [\lambda e^{-\lambda t} \otimes 1] g_c(t) = [1-e^{-\lambda t}] g_c(t)
\]
\[
q_{43}(t) = e^{-\lambda t} w(t) ;
\]
\[
q_{45}(t) = \lambda e^{-\lambda t} \quad \overline{W}(t)
\]
\[
q_{53}(t) = g_c(t) ;
\]
\[
q_{61}(t) = p_1 \lambda e^{-\lambda t}
\]
\[
q_{62}(t) = q_1 \lambda e^{-\lambda t} \quad (5.2.1-5.2.17)
\]

The non-zero elements \( p_{ij} \) are

\[
p_{01} = 1, \quad p_{10} = p E_1^*(\lambda), \quad p_{12} = q E_1^*(\lambda)
\]
Fig. 5.2.
\[ p_{13} = E_2^*(\lambda), p_{14} = (1-a) E_1^*(\lambda), p_{15} = \lambda E_3^*(\lambda) \]
\[ p_{21} = h_1^*(\lambda), p_{25} = 1-h_1^*(\lambda), p_{30} = p_{1g_c}^*(\lambda) \]
\[ p_{36} = q_{1g_c}^*(\lambda), p_{37} = 1-g_c^*(\lambda), p_{33}^{(7)} = 1-g_c^*(\lambda) \]
\[ p_{43} = w^*(\lambda), p_{45} = 1-w^*(\lambda), p_{53} = 1, \]
\[ p_{61} = p_1, p_{62} = q_1 \quad (5.2.18-5.2.34) \]

By these transition probabilities, it can be verified that
\[ p_{01} = 1, \quad p_{10} + p_{12} + p_{13} + p_{14} + p_{15} = 1, \quad p_{21} + p_{25} = 1 \]
\[ p_{30} + p_{36} + p_{37} = 1, \quad p_{30} + p_{36} + p_{33}^{(7)} = 1 \]
\[ p_{43} + p_{45} = 1, \quad p_{53} = 1, \quad p_{61} + p_{62} = 1 \quad (5.2.35-5.2.42) \]

The mean sojourn times \( \mu_i \) are:
\[ \mu_0 = \frac{1}{\lambda}, \quad \mu_1 = E_3^*(\lambda), \quad \mu_2 = -h_1^*(0) \]
\[ \mu_3 = \frac{1-g_c^*(\lambda)}{\lambda}, \quad \mu_4 = \frac{1-w^*(\lambda)}{\lambda} \]
\[ \mu_5 = -g_c^*(0), \quad \mu_6 = \frac{1}{\lambda}. \quad (5.2.43-5.2.49) \]

The unconditional mean time by the system to transition for any state \( j \) when it is counted from the epoch of entrance into state \( i \) is mathematically stated as:
\[ m_{ij} = \int_0^\infty q_{ij}(t)dt = -q_{ij}^*(0) \quad (5.2.50) \]

Thus
\[ m_{01} = \mu_0; \quad m_{10} + m_{12} + m_{13} + m_{14} + m_{15} = \mu_1 \]
\[ m_{21} + m_{25} = \mu_2; \quad m_{30} + m_{36} + m_{37} = \mu_3 \]
\[ m_{30} + m_{36} + m_{33}^{(7)} = \mu_5; \quad m_{43} + m_{45} = \mu_4 \]
\[ m_{53} = \mu_5; \quad m_{61} + m_{62} = \mu_6 \quad (5.2.51-5.2.58) \]
Mean Time to System Failure

By probabilistic arguments, we obtain the following recursive relations for \( \phi_i(t) \):

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t)\phi_1(t) \\
\phi_1(t) &= Q_{10}(t)\phi_0(t) + Q_{12}(t)\phi_2(t) + Q_{13}(t)\phi_3(t) + Q_{14}(t)\phi_4(t) + Q_{15}(t) \\
\phi_2(t) &= Q_{21}(t)\phi_1(t) + Q_{25}(t) \\
\phi_3(t) &= Q_{30}(t)\phi_0(t) + Q_{36}(t)\phi_6(t0 + Q_{37}(t) \\
\phi_4(t) &= Q_{43}(t)\phi_3(t) + Q_{45}(t) \\
\phi_6(t) &= Q_{61}(t)\phi_1(t) + Q_{62}(t)\phi_2(t)
\end{align*}
\]

Taking L.S.T. of these relations and solving them for \( \phi_0**(s) \), we obtain

\[
\phi_0**(s) = \frac{N(s)}{D(s)}
\]

where

\[
\begin{align*}
N(s) &= Q_{01}**(s)[Q_{15}**(s)+Q_{12}**(s)Q_{25}**(s)+Q_{14}**(s)Q_{45}**(s) + \{Q_{13}**(s)+Q_{14}**(s)Q_{43}**(s)Q_{36}**(s)Q_{62}**(s)Q_{25}**(s) + Q_{37}**(s)\}] \\
D(s) &= 1 - Q_{01}**(s)Q_{10}**(s) - Q_{12}**(s)Q_{21}**(s) - [Q_{13}**(s) \nonumber \\
&+ Q_{14}**(s)Q_{43}**(s)[Q_{36}**(s)Q_{61}**(s)+Q_{62}**(s)Q_{21}**(s)] + Q_{30}**(s)Q_{01}**(s)\}
\end{align*}
\]

Now, the mean time to system failure (MTSF) when the system starts from the state ‘0’ is

\[
T_0 = \lim_{s \to 0} \frac{1 - \phi_0**(s)}{s} = \frac{N}{D}
\]
\[ D = p_{15} + p_{14} p_{45} + p_{12} p_{25} + (p_{13} + p_{14} p_{43}) [ p_{36} p_{62} p_{25} + p_{37} ] \] (5.2.69–6.2.70)

**Availability Analysis**

The availability \( A_i(t) \) is seen to satisfy the following recursive relations:

\[
\begin{align*}
A_0(t) &= M_0(t) + q_{01}(t) A_1(t) \\
A_1(t) &= M_1(t) + q_{10}(t) A_0(t) + q_{12}(t) A_2(t) + q_{13}(t) A_3(t) \\
&\quad + q_{14}(t) A_4(t) + q_{15}(t) A_5(t) \\
A_2(t) &= M_2(t) + q_{21}(t) A_1(t) + q_{25}(t) A_5(t) \\
A_3(t) &= M_3(t) + q_{30}(t) A_0(t) + q_{36}(t) A_6(t) + q_{33}(t) A_3(t) \\
A_4(t) &= M_4(t) + q_{43}(t) A_3(t) + q_{45}(t) A_5(t) \\
A_5(t) &= q_{53}(t) A_3(t) \\
A_6(t) &= M_6(t) + q_{61}(t) A_1(t) + q_{62}(t) A_2(t) \\
\end{align*}
\] (5.2.71–5.2.77)

where

\[
\begin{align*}
M_0(t) &= e^{-i\lambda t}, \quad M_1(t) = e^{-i\lambda t} E_1(t), \quad M_2(t) = e^{-i\lambda t} \bar{W}_1(t) \\
M_3(t) &= e^{-i\lambda t} \bar{G}_c(t), \quad M_4(t) = e^{-i\lambda t} \bar{W}(t), \quad M_6(t) = e^{-i\lambda t} \\
\end{align*}
\] (5.2.78–5.2.83)

Taking L.T. of the above equations and solving them for \( A_0^*(s) \), we have

\[ A_0^*(s) = N_1(s) / D_1(s) \] (5.2.84)

where

\[ \begin{align*}
N_1(s) &= [1 - q_{33}(7)^*(s)] [M_0^*(s)(1 - q_{12}(s)q_{21}(s)) + q_{01}(s)(q_{12}(s)M_2^*(s) \\
&\quad + q_{14}(s)M_4^*(s) + M_1^*(s))] - [(q_{13}(s) + q_{15}(s)q_{53}^*(s)) \\
&\quad + q_{14}(s)(q_{43}^*(s) + q_{45}(s)q_{53}^*(s))] [q_{36}(s)M_0^*(s)q_{61}^*(s) \\
&\quad + q_{62}(s)q_{21}^*(s) - q_{01}(s)] - q_{01}(s)[M_3^*(s) + q_{36}(s)q_{62}^*(s)M_2^*(s)] \\
&\quad - q_{25}(s)q_{53}(s)q_{36}(s)[M_0^*(s)q_{61}(s)q_{12}(s) + q_{62}(s)]
\end{align*} \]
The steady state availability of the system is given by

\[ A_0 = \frac{N_1}{D_1} \quad (5.2.87) \]

where

\[ N_1 = (p_{30} + p_{36})[(1-p_{12}p_{21}) \mu_0 + \mu_1 + p_{12}\mu_2 + p_{14}\mu_4] + (p_{13} + p_{14} + p_{15})[\mu_3 + p_{36}p_{62}\mu_2 + p_{25}\mu_0] - p_{36}p_{62}p_{25}[(1-p_{12})\mu_0 + \mu_1 + p_{14}\mu_4] \]

\[ D_1 = [p_{30}(1-p_{12}p_{21}) + p_{10}p_{36}(1-p_{62}p_{25})] \mu_0 + [p_{30} + p_{36}(1-p_{25}p_{62})]\mu_1 + [p_{12}(p_{30} + p_{36}) + p_{36}p_{62}(p_{13} + p_{14} + p_{15})]\mu_2 + (1-p_{10} - p_{12}p_{21})\mu_5 + p_{36}\mu_0 + [p_{36}(p_{61} + p_{62}p_{21}) + p_{30}][p_{14}\mu_4 + (p_{15} + p_{14}\mu_3)\mu_5] + p_{25}(p_{10}p_{36}p_{62} + p_{30}p_{12} + p_{36}p_{61}p_{12} + p_{36}p_{62})\mu_5 \quad (5.2.88-5.2.89) \]

**Busy Period Analysis of the Expert Repairman**

By probabilistic arguments, we have the following recursive relations for \( B_i^c(t) \):-

\[ B_0^c(t) = q_{01}(t) \oplus B_1^c(t) \]

\[ B_1^c(t) = q_{10}(t) \oplus B_0^c(t) + q_{12}(t) \oplus B_2^c(t) + q_{13}(t) \oplus B_3^c(t) + q_{14}(t) \oplus B_4^c(t) \]

\[ B_2^c(t) = q_{21}(t) \oplus B_1^c(t) + q_{25}(t) \oplus B_3^c(t) \]

\[ B_3^c(t) = W_3(t) + q_{30}(t) \oplus B_0^c(t) + q_{36}(t) \oplus B_6^c(t) + q_{33}^7(t) \oplus B_3^c(t) \]

\[ B_4^c(t) = q_{43}(t) \oplus B_3^c(t) + q_{45}(t) \oplus B_5^c(t) \]

\[ B_5^c(t) = W_5(t) + q_{53}(t) \oplus B_3^c(t) \]
\[ B_6^c(t) = q_{61}(t)\overline{B}_1^c(t) + q_{62}(t)\overline{B}_2^c(t) \]  
(5.2.90–5.2.96)

where

\[ W_3(t) = W_5(t) = \overline{G}_c(t) \]  
(5.2.97)

Taking L.T. of the above equations and solving them for \( B_0^*(s) \), we have

\[ B_0^*(s) = N_2(s)/D_1(s) \]  
(5.2.98)

where

\[ N_2(s) = q_{01}^*(s)W_5^*(s)[\{1-q_{33}^{(7)}(s)\} \{q_{12}^*(s)q_{25}^*(s) + q_{14}^*(s)q_{45}^*(s) + q_{15}^*(s)\} + q_{13}^*(s)q_{33}^*(s) + q_{14}^*(s)q_{45}^*(s)q_{53}^*(s)] + q_{25}^*(s)q_{62}^*(s)(q_{13}^*(s) + q_{14}^*(s)q_{43}^*(s)) + q_{12}^*(s)q_{53}^*(s) \]  
(5.2.99)

In steady state, the total fraction of the time for which the system is under repair of the expert is given by

\[ B_0^c = N_2/D_1 \]  
(5.2.100)

where

\[ N_2 = \mu_5[p_{12}p_{25}p_{53} + (p_{13} + p_{14} + p_{15})(1 + p_{36}p_{62}p_{25}) + (p_{30} + p_{36})] \]  
(5.2.101)

**Expected Number of Visits by the Expert Repairman**

By probabilistic arguments, we have the following relations for \( V_j^c(t) \):

\[ V_0^c(t) = Q_{01}(t)\overline{S}V_1^c(t) \]
\[ V_1^c(t) = Q_{10}(t)\overline{S}V_0^c(t) + Q_{12}(t)\overline{S}V_2^c(t) + Q_{13}(t)\overline{S}[1 + V_3^c(t)] + Q_{14}(t)\overline{S}V_4^c(t) + Q_{15}(t)\overline{S}[1 + V_5^c(t)] \]
\[ V_2^c(t) = Q_{21}(t)\overline{S}V_1^c(t) + Q_{25}(t)\overline{S}[1 + V_5^c(t)] \]
\[ V_3^c(t) = Q_{30}(t)\overline{S}V_0^c(t) + Q_{36}(t)\overline{S}V_6^c(t) + Q_{33}^{(7)}(t)\overline{S}V_3^c(t) \]
\[ V_4^c(t) = Q_{43}(t)\overline{S}[1 + V_3^c(t)] + Q_{45}(t)\overline{S}[1 + V_5^c(t)] \]
\[ V_5^c(t) = Q_{53}(t)\overline{S}V_3^c(t) \]
\[ V_0^c(t) = Q_{61}(t)S V_1^c(t) + Q_{62}(t)S V_2^c(t) \quad (5.2.102-5.2.108) \]

Taking L.S.T. of the above equations and solving them for \( V_0^{c**}(s) \), we have

\[ V_0^{c**}(s) = N_3(s)/D_1(s) \quad (5.2.109) \]

where

\[ N_3(s) = q_{01}\{q_{36}*(s)q_{62}*(s)q_{25}*(s)\{q_{13}*(s)+q_{14}*(s)q_{43}*(s)\}\{1-q_{33}*(s)\} \\
+ (1-q_{33}*(s))\{q_{14}*(s)(q_{43}*(s)+q_{45}*(s))+q_{13}*(s)+q_{15}*(s) \\
+ q_{12}*(s)q_{25}*(s)\}\} \quad (5.2.110) \]

In steady state, the number of visits per unit time of the expert repairman is given by

\[ V_0^c = N_3/D_1 \quad (5.2.111) \]

where

\[ N_3 = (p_{30} + p_{36}) (1-p_{10}-p_{12}p_{21}) \quad (5.2.112) \]

**Profit Analysis**

The expected profit incurred to the system in steady state is given by

\[ P_2 = C_0A_0 - C_1B_0^c - C_3V_0^c - C_5 \quad (5.2.113) \]

where

\( C_0, C_1, C_3 \) and \( C_5 \) are same as defined for Model 1 of the chapter.

**Particular Case**

Let us assume that

\[ g(t) = \alpha e^{-\mu t}, \quad g_c(t) = \alpha_1 e^{-\mu_1 t}, \quad i(t) = \gamma e^{-\eta t}, \quad h_1(t) = \beta e^{-\beta_1 t} \]

Therefore, we have

\[ p_{01} = 1, \quad p_{10} = \frac{p\alpha}{\alpha + \gamma + \lambda}, \quad p_{12} = \frac{q\alpha}{\alpha + \gamma + \lambda} \]

\[ p_{13} = \frac{\gamma}{\alpha + \gamma + \lambda}, \quad p_{14} = \frac{(1-a)\alpha}{\alpha + \gamma + \lambda}, \quad p_{15} = \frac{\lambda}{\alpha + \gamma + \lambda} \]
\[ p_{21} = \frac{\beta_1}{\beta_1 + \lambda}, \quad p_{25} = \frac{\lambda}{\beta_1 + \lambda}, \quad p_{30} = \frac{p_{1} \alpha_1}{\alpha_1 + \lambda} \]

\[ p_{36} = \frac{\alpha_1}{\alpha_1 + \lambda}, \quad p_{37} = \frac{\lambda}{\alpha_1 + \lambda}, \quad p_{33}^{(7)} = \frac{\lambda}{\alpha_1 + \lambda} \]

\[ p_{43} = \frac{\beta}{\beta + \lambda}, \quad p_{45} = \frac{\lambda}{\beta + \lambda}, \quad p_{53} = 1 \]

\[ p_{61} = p_{1}, \quad p_{62} = q_{1} \]

\[ \mu_0 = \frac{1}{\lambda}, \quad \mu_1 = \frac{1}{\alpha + \gamma + \lambda}, \quad \mu_2 = \frac{1}{\beta_1}, \quad \mu_3 = \frac{1}{\alpha_1 + \lambda} \]

\[ \mu_4 = \frac{1}{\beta + \lambda}, \quad \mu_5 = \frac{1}{\alpha}, \quad \mu_6 = \frac{1}{\lambda} \quad (5.2.114-5.2.130) \]

Using the above equations and the equations (5.2.68), (5.2.87), (5.2.100), (5.2.111) and (5.2.113), the expressions for the MTSF and the profit can be obtained for this particular case.

**Graphical Interpretation**

The above particular case is considered for the graphical representation.

Figs. 5.5 and 5.6 shows the relationship between MTSF vs failure rate and profit (\(P_2\)) vs failure rate respectively. Change in their values is also plotted for variation in the values of patience rate (\(\gamma\)). It is observed from the graphs that the MTSF and the profit both decrease with increase in the values of failure rate. However, their values become higher for the higher values of patience rate.

**Comparison Between Model 1 and Model 2**

We have compared the profits of the two models graphically for the particular cases already taken.
Fig. 5.7 depicts the behaviour of the difference of profits $(P_2-P_1)$ vs cost $(C_3)$ per visit of the expert for variation in the values of patience rate $(\gamma)$. It is clear from the graph that the difference $(P_2-P_1)$ decreases with increase in the values of $(C_3)$. It decreases more rapidly for lower values of patience rate $(\gamma)$ as can be seen from the graph that initially, $P_2-P_1$ is higher for lower value of patience rate but its feature reverses i.e. $P_2-P_1$ becomes lower for lower value of patience rate beyond a certain value of cost $(C_3)$.

Following can be observed from the graph:

(i) If $\gamma = 1$, though $P_2-P_1$ has the highest value initially but this value becomes lowest as compared to the values of $\gamma = 3, 5 & 7$ when $C_3 = 430$. So, $P_2-P_1$ decreases more rapidly w.r.t. cost $(C_3)$ for lower values of patience rate.

(ii) If $\gamma = 1; \; P_2-P_1 > 0 \; or \; = 0 \; or \; < 0$ according as $C_3 < 1500 \; or \; = 1500 \; or \; > 1500$ which implies that Model 2 is better than Model 1 if $C_3 < 1500$, the two models are equally good if $C_3 = 1500$, Model 1 is better than Model 2 if $C_3 > 1500$.

(iii) If $\gamma = 3; \; P_2-P_1 > 0 \; or \; = 0 \; or \; < 0$ according as $C_3 < 2200 \; or \; = 2200 \; or \; > 2200$ which implies that Model 2 is better than Model 1 if $C_3 < 2200$, the two models are equally good if $C_3 = 2200$, Model 1 is better than Model 2 if $C_3 > 2200$.

(iv) For $\gamma = 5$ or 7, similar interpretation can be given as given for $\gamma = 1$ or 3. The difference in interpretation would be only of the values of $C_3$ at which $P_2-P_1 = 0$ which are 2950 and 4050 for $\gamma = 5$ and 7 respectively.
(v) It is also interpreted that for lower values of patience rate, \( P_2 - P_1 \) becomes 0 more rapidly for lower values of cost \((C_3)\).

Fig. 5.8 shows the behaviour of the difference of profits \((P_2 - P_1)\) vs cost \((C_3)\) for variation in the values of waiting rate \((\beta)\). It is observed from the graph that the trend for \( P_2 - P_1 \) is decreasing with increase in the values of \( C_3 \). But, here initially, \( P_2 - P_1 \) is higher for higher values of waiting rate which after certain value of cost \((C_3)\) i.e. after crossing the value of \( C_3 = 1050 \), its nature reverses and becomes lower for higher values of waiting rate. Following observations are had:

(i) If \( \beta_1 = 3 \); \( P_2 - P_1 > 0 \) or \( = 0 \) or \( < 0 \) according as \( C_3 < 1650 \) or \( = 1650 \) or \( > 1650 \) i.e. Model 2 is better than Model 1 if \( C_3 < 1650 \), the two models are equally good if \( C_3 = 1650 \); Model 1 is better than Model 2 if \( C_3 > 1650 \).

(ii) If \( \beta_1 = 7 \), \( P_2 - P_1 > 0 \) or \( = 0 \) or \( < 0 \) according as \( C_3 < 1525 \) or \( = 1525 \) or \( > 1525 \) i.e. Model 2 is better than Model 1 if \( C_3 < 1525 \), the two models are equally good if \( C_3 = 1525 \), Model 1 is better than Model 2 if \( C_3 > 1525 \).
MTSF VERSUS FAILURE RATE ($\lambda$) FOR DIFFERENT VALUES OF PATIENCE RATE ($\gamma$)

$a=0.7$, $b=0.5$, $\alpha=2$, $\alpha_1=3$, $\alpha_2=2$, $\beta=5$, $p_1=0.5$, $q_1=0.5$

Fig. 5.3
PROFIT VERSUS FAILURE RATE ($\lambda$) FOR DIFFERENT VALUES OF PATIENCE RATE ($\gamma$)

- $a=0.7$, $b=0.5$, $\alpha_1=2$, $\alpha_2=3$, $\alpha_3=2$, $\beta=5$, $p_1=0.5$, $q_1=0.5$
- $C_0=1000$, $C_1=100$, $C_2=50$, $C_3=50$, $C_4=30$, $C_5=50$

Fig. 5.4
MTSF VERSUS FAILURE RATE ($\lambda$) FOR DIFFERENT VALUES OF PATIENCE RATE ($\gamma$)

$$a=0.7, b=0.5, \alpha=2, \alpha_i=3, \beta=5, \beta_i=5, p_i=0.5, q_i=0.5$$

Fig. 5.5
PROFIT VERSUS FAILURE RATE ($\lambda$) FOR DIFFERENT VALUES OF PATIENCE RATE ($\gamma$)

- \(a=0.7, b=0.5, \alpha=2, \alpha_1=3, \beta=5, \beta_1=5, p_1=0.5, q_1=0.5\)
- \(C_0=1000, C_1=100, C_3=50, C_5=50\)

Fig. 5.6
DIFFERENCE OF PROFITS ($P_2 - P_1$) VERSUS COST ($C_3$) FOR DIFFERENT VALUES OF PATIENCE RATE ($\gamma$)

$a=0.7$, $b=0.5$, $\alpha_1=2$, $\alpha_2=3$, $\alpha_3=2$, $\beta=5$, $\beta_1=5$, $p_1=0.5$, $q_1=0.5$, $C_0=1000$, $C_1=100$, $C_2=50$, $C_3=50$, $C_4=30$, $C_5=50$

Fig. 5.7
DIFFERENCE OF PROFITS ($P_2 - P_1$) VERSUS COST ($C_3$) FOR DIFFERENT VALUES OF WAITING RATE ($\beta_1$)

Fig. 5.8

Cost ($C_3$)

$\beta_1=3$

$\beta_1=7$

$$a=0.7, \ b=0.5, \ \alpha_1=2, \ \alpha_2=3, \ \alpha_2=2, \ \beta=5, \ \gamma=1,$$

$$p_1=0.5, \ q_1=0.5, \ C_0=1000, \ C_1=100,$$

$$C_2=50, \ C_3=50, \ C_4=30, \ C_5=50$$