CHAPTER 6

SINGLE LAYERED CABLE UNDER CONSTRAINED BENDING - DEVELOPMENT OF NEW MATHEMATICAL MODEL AND VALIDATION

6.1 INTRODUCTION

The main objectives of this quasi-static test were to observe the bending response of stranded cable and to determine their Moment-curvature relationships under various applied transverse loads. The model can also predict the end response of the global strand axial force and strand twisting moment for various strand curvatures due to applied transverse loading.

6.2 ASSUMPTIONS AND METHOD OF APPROACH

For developing the new thin rod model, a simple strand with a core wire and one layer of six wires with a core-wire type contact, was deliberately chosen for comparison with the models considered in the literature. The contacts were assumed to be in unlimited coulomb friction. As the cable runs over a drum or pulley, the cable undergoes a constant curvature bending. The values of deformation, wire curvatures and torsion are considered for this scenario. The end conditions of the strand are classified into two categories: fixed end and free end conditions. Both ends of the strand were as fixed-fixed considered in this model. In this condition the model can be restricted to rotation about and translation along the strand axis. The flat drum surface is considered for the present construction and at any instant one of the six wires
in the outer layer only establishes contact with the drum. Unlike the rest of the five wires, this wire contacting the drum will experience a force and a couple from the drum additionally. This is referred as line load per unit length and twisting couple per unit length respectively in the present formulation. So the five wires will be treated as free bending and the single contacting wire will be treated as constrained bending in the formulation of the global stiffness of the strand. The new discrete thin rod model was developed with the following assumptions. The core was assumed to be radially rigid and the Poisson’s effects of the core and wires were not considered.

6.3 DEVELOPMENT OF THE GENERAL MATHEMATICAL MODEL

The model was further developed and derived from the previous chapter 4, with the inclusion of line load per unit length and twisting couple per unit length from the pulley as shown in Figure 6.1. The model can predict the global strand response of the strand axial force, strand twisting moment and strand bending moment for comparison with the FEA model. The deformed wire curvatures and twist of the wire have been adopted from the previous chapter 4.

Figure 6.1 Simple stranded cable wrapped around pulley Courtesy: Costello (1990)
6.3.1 Constitutive Equations

When the stranded cable runs over a drum, only one wire from the outer layer of the present configuration makes contact with the drum at any instant. Unlike the rest of the wires in the outer layer, this wire (bottom wire) will experience an additional force and moment from the drum as shown in Figure 6.1. Let the radial force ‘\( p \)’ be the distributed load per unit length which the drum exerts on the contacting wire with the drum. This force in conjunction with the friction coefficient ‘\( \mu \)’ between the stranded cable and the pulley, gives rise to a friction force ‘\( \mu p \)’, which modifies the wire tension as:

\[
T_{\text{wire}} = EA \sin^2 \alpha \sigma + EAr \sin \alpha \cos \alpha \frac{\delta \gamma}{h} + \frac{EAr \sin^2 \alpha \cos \phi}{\rho} + \mu p \rho 
\]  

(6.1)

The internal friction acting between the wires and the core is sufficient to prevent any relative slip happening between them, especially when the strand is much below the critical curvature.

Figure 6.2 cross-section of the strand over the pulley at one incident
Let ‘q’ be the twisting couple per unit length applied to the strand by the drum along the binormal direction. This modifies the shear force in the binormal direction of the wire contacting the drum as:

\[ N'_{\text{wire}} = H\kappa' - G'\tau_0 + q \]  

(6.2)

The ‘p’ and ‘q’ are applied to the strand by the drum in order to maintain the equilibrium of the strand. Figure 6.2 shows the cross section of the cable indicating the position of the wire (\(\phi\)) with respect to the vertical axis. The other equations of wire shear forces (\(N\) & \(N'\)), wire twisting moment (\(H\)) and wire bending moment (\(G\) & \(G'\)) are adopted from the previous chapter 4.

6.3.2 Equilibrium Equations of Stranded Cable

The resultant external axial force, twisting moment and bending moment of the stranded cable have the wire components from Equations (6.1) & (6.2) and Equations (4.12) to (4.16). All the forces on the core and the wires are projected along the axial direction of the strand which is given by Equation 6.3

\[ F_x = E_c A_c + (m - 1) \left( T \sin \alpha + N' \cos \alpha \right) \\
+ T_{\text{wire}} \sin \alpha + N'_{\text{wire}} \cos \alpha \]  

(6.3)

The total axial twisting moment acting on the central wire and the surrounding wires are given by Equation 6.4

\[ M_x = C_c J_c + (m - 1) \left( H \sin \alpha + G' \cos \alpha + T_r \cos \alpha - N' r \sin \alpha \right) \\
+ \left( H \sin \alpha + G' \cos \alpha + T_{\text{wire}} r \cos \alpha - N'_{\text{wire}} r \sin \alpha \right) \]  

(6.4)

The bending moment of the stranded cable which has been determined earlier using the equilibrium approach by Sathikh et al (2000) was modified accordingly for the constrained bending scenario as in Equation 6.5.
\[ M_b = E_c I_c + (m - 1) \left( T \sin \alpha + N' \cos \alpha r \cos \phi + G \sin \phi \right) \]
\[ + G' \sin \alpha \cos \phi - H \cos \alpha \cos \phi \]
\[ - N \cot \alpha r \sin \phi \]
\[ \left( T_{wire} \sin \alpha + N'_{wire} \cos \alpha r \cos \phi + G \sin \phi \right) \]
\[ + G' \sin \alpha \cos \phi - H \cos \alpha \cos \phi \]
\[ - N \cot \alpha r \sin \phi \]

(6.5)

6.3.3 Stiffness of Stranded Cable

The Stranded cable global stiffness is derived using the stiffness matrix, relating to the global strand loads to deformations.

\[
\begin{pmatrix}
F_a \\
M_t \\
M_b
\end{pmatrix} =
\begin{pmatrix}
K_{aa} & K_{at} & K_{ab} \\
K_{ta} & K_{tt} & K_{tb} \\
K_{ba} & K_{bt} & K_{bb}
\end{pmatrix}
\begin{pmatrix}
\varepsilon \\
\delta\chi/h \\
1/\rho
\end{pmatrix}
\]

(6.6)

\(F_a, M_t\) and \(M_b\) are the strand tensile, torsion and bending loads respectively. Likewise \(\varepsilon, \delta\chi/h\) and \(1/\rho\) are the strand axial strain, rotation per unit length and curvature respectively. \(K_{aa}, K_{at}, K_{bb}\) are effective strand axial stiffness, strand torsional rigidities and flexural rigidities respectively. \(K_{at}, K_{tt}\) are the tension – torsion, \(K_{ab}, K_{bt}\) are the tension – bending and \(K_{tb}, K_{bt}\) are the torsion – bending coupling parameters respectively.

\[
\begin{pmatrix}
F_a \\
M_t \\
M_b
\end{pmatrix} =
\begin{pmatrix}
K_{aa}(5\text{ wires-core}) & K_{at}(5\text{ wires}) & K_{ab}(5\text{ wires}) \\
K_{ta}(5\text{ wires}) & K_{tt}(5\text{ wires-core}) & K_{tb}(5\text{ wires}) \\
K_{ba}(5\text{ wires}) & K_{bt}(5\text{ wires}) & K_{bb}(5\text{ wires-core})
\end{pmatrix}
\begin{pmatrix}
\varepsilon \\
\delta\chi/h \\
1/\rho
\end{pmatrix}
\]

(6.7)

The stiffness coefficients were derived together for the core and the five wires of the outer layer (pure bending) and the wire contacting with drum
(constrained bending). The global stiffness of the assembly of the strand is given in Equation (6.7).

The stiffness coefficients of 5 wires and the single wire are given by

\[
K_{\text{as}(5\ \text{wires, core})} = E_c A_c + m \left( \frac{E A \sin^3 \alpha + CJ \sin \alpha \cos^6 \alpha}{r^2} \right) + \frac{E I \sin^3 \alpha \cos^4 \alpha}{r^2} \right) \right) \right)
\]

\[
K_{\text{as}(5\ \text{wires})} = m \left( \frac{E A \sin^2 \alpha \cos \alpha + CJ \sin^4 \alpha \cos^3 \alpha}{r^2} \right) \right) \right) \right)
\]

\[
K_{\text{as}(5\ \text{wires, core})} = C_c J_c + m \left( \frac{C J \sin^5 \alpha + E I \sin \alpha \cos^2 \alpha (1 + \sin^2 \alpha) \right)
\]

\[
K_{\text{as}(5\ \text{wires})} = \left( \sum_{n=1}^{2} \cos \frac{n\pi}{3} + \sum_{n=4}^{6} \cos \frac{n\pi}{3} \right) \left( \frac{G J \sin^4 \alpha \cos \alpha}{r} \right) + \frac{E A r^2 \sin \alpha \cos^2 \alpha}{r} \right) \right) \right)
\]

\[
K_{\text{as}(5\ \text{wires})} = \left( \sum_{n=1}^{2} \cos \frac{n\pi}{3} + \sum_{n=4}^{6} \cos \frac{n\pi}{3} \right) \left( \frac{-G J \sin^4 \alpha \cos \alpha}{r} \right) + \frac{E I \sin^4 \alpha \cos \alpha}{r} + \frac{E A r^2 \sin^2 \alpha \cos \alpha}{r} \right) \right) \right)
\]
The equations for stiffness coefficients from (6.8) to (6.16) were derived for 5 wires in a layer and for the core wire which are treated as free bending case.
\[
K_{\text{air wire}} = \frac{\left(\frac{EA \sin^3 \alpha + CJ \sin \alpha \cos^6 \alpha}{r^2} + \frac{EI \sin^3 \alpha \cos^4 \alpha}{r^2}\right) \left[1 + \frac{r \sin \alpha}{\rho}\right]}{\left[1 - \mu \sin \alpha\right] \left[1 + \frac{r \sin \alpha}{\rho}\right] - \left(\frac{4r \cos^2 \alpha}{\rho}\right)} \tag{6.17}
\]

\[
K_{\text{air wire}} = \frac{\left(\frac{EAr \sin^2 \alpha \cos \alpha + CJ \sin^4 \alpha \cos^3 \alpha}{r} - \frac{EI \sin^2 \alpha \cos^3 \alpha (1 + \sin^2 \alpha)}{r}\right) \left[1 + \frac{r \sin \alpha}{\rho}\right]}{\left[1 - \mu \sin \alpha\right] \left[1 + \frac{r \sin \alpha}{\rho}\right] - \left(\frac{4r \cos^2 \alpha}{\rho}\right)} \tag{6.18}
\]

\[
K_{\text{air wire}} = \frac{\left(\frac{EAr \sin^3 \alpha \cos 180^\circ}{r} - \frac{CJ \sin^3 \alpha \cos^4 \alpha \cos 180^\circ}{r}\right) \left[1 + \frac{r \sin \alpha}{\rho}\right]}{\left[1 - \mu \sin \alpha\right] \left[1 + \frac{r \sin \alpha}{\rho}\right] - \left(\frac{4r \cos^2 \alpha}{\rho}\right)} \tag{6.19}
\]
\[
K_{at(1\ wire)} = \frac{\left( C J \sin^2 \alpha \cos^3 \alpha - \frac{E I \sin^2 \alpha \cos^3 \alpha}{r} \right)}{1 + \frac{r \sin \alpha}{\rho^2}}
+ \left( \frac{\mu r \cos \alpha}{K_{at(1\ wire)}} \right)
\]

\[
K_{ar(1\ wire)} = \frac{\left( C J \sin^5 \alpha + \frac{E I \sin \alpha \cos^2 \alpha\left( I + \sin^2 \alpha \right)}{r} + E A r^2 \sin ^2 \alpha \cos^2 \alpha - C J \sin^5 \alpha \cos^2 \alpha \right)}{1 + \frac{r \sin \alpha}{\rho^2}}
+ \left( \frac{\mu r \cos \alpha}{K_{ar(1\ wire)}} \right)
\]

\[
K_{rh(1\ wire)} = \frac{\left( - C J \sin^4 \alpha \cos^2 \alpha \cos \alpha \cos 180^\circ + E I \sin^4 \alpha \cos^2 \alpha \left( I + \sin^2 \alpha \right) \right)}{1 + \frac{r \sin \alpha}{\rho^2}}
+ \left( \frac{\mu r \cos \alpha}{K_{rh(1\ wire)}} \right)
\]

\[
K_{hr(1\ wire)} = b a_{180} + \mu \sin \alpha r \cos 180^\circ K_{at(1\ wire)} + \frac{\cos \alpha r \cos 180^\circ}{\rho} K_{at(1\ wire)}
\]

\[
K_{hr(1\ wire)} = b t_{180} + \mu \sin \alpha r \cos 180^\circ K_{at(1\ wire)} + \frac{\cos \alpha r \cos 180^\circ}{\rho} K_{at(1\ wire)}
\]

\[
K_{ab(1\ wire)} = b h_{180} + \mu \sin \alpha r \cos 180^\circ K_{ah(1\ wire)} + \frac{\cos \alpha r \cos 180^\circ}{\rho} K_{ah(1\ wire)}
\]

\[
b a_{180} = \frac{\sin \alpha \cos 180^\circ}{r} \left( E A r^2 \sin^2 \alpha + C J \cos^6 \alpha + E I \sin^2 \alpha \cos^4 \alpha \right)
- E I \sin^2 \alpha \cos^2 \alpha - C J \cos^4 \alpha
\]
The equations for stiffness coefficients from (6.17) to (6.28) were derived for a single wire of the outer layer which interacts with the drum. Hence the line load per unit length and twisting couple per unit length (drum effect) has been incorporated.

6.4 DEVELOPMENT OF THE FINITE ELEMENT MODEL

The finite element model constructed for the single layer free bending in chapter 4 has been utilized for the present analysis. Additionally a sector of the drum was modeled using brick elements and was given a very high Young’s modulus (2.00E+005 N/mm$^2$) to make the drum free from any deformation. The loading conditions to simulate constrained bending are as follows:

The master nodes at the either ends of the strand were fully clamped by constraining all degrees of freedom (d.o.f) of the nodes except for rotation about x axis. Then, a pressure was applied on the drum which is located on the mid span of the cable in small increments and the corresponding cable response was recorded. The Figure 6.3 shows the finite element model of the cable and the drum at pre-constrained bending stage.
Figure 6.3  FE model of strand and flat drum representing constrained bending loading

The Figure 6.4 shows the detailed view of the various elements of this assembly. Mesh convergence has been tested and care has been taken to represent finer mesh at areas where there exists interactions between the elements.

Figure 6.4 Three dimensional view of the FE model (a) strand (b) drum.

6.5  RESULTS AND DISCUSSION

The geometrical data of the 1x7 wire strand has been adopted from LeClair and Costello (1988) to study the bending response. The numerical data of the strand considered is presented in Table 6.1.
Table 6.1  Geometrical parameters and mechanical properties of 1x7 wire strand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of helical wires</td>
<td>$m$</td>
<td>6</td>
</tr>
<tr>
<td>Radius of core</td>
<td>$R_c$</td>
<td>0.787 mm</td>
</tr>
<tr>
<td>Radius of helical wire</td>
<td>$R_w$</td>
<td>0.737 mm</td>
</tr>
<tr>
<td>Helix angle</td>
<td>$\alpha$</td>
<td>70°</td>
</tr>
<tr>
<td>Young’s modulus for core and wire</td>
<td>$E_c$ &amp; $E_w$</td>
<td>$207 \times 10^3$ N/mm$^2$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Position angle of helical wires strand</td>
<td>$\phi$</td>
<td>0° to 360°</td>
</tr>
</tbody>
</table>

The present work predicts the pre-slip response of the strand under constrained bending in the presence of an external agency, drum. The effect of wire deformation on the wire curvatures and twist due to strand bending is considered. The appropriate kinematic variables were derived using Serret-Frenet equations, in order to obtain the constitutive equations.

As there has been no mathematical model addressed in the literature on the response of the stranded cable under constrained bending, an attempt is made here to study this with the present model of free bending case.
Table 6.2 Comparison of stiffness coefficients between present analytical model of free bending and constrained bending

<table>
<thead>
<tr>
<th>Stiffness co-efficient</th>
<th>Present Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Free bending</td>
</tr>
<tr>
<td>$K_{aa}$ (N)</td>
<td>2162919</td>
</tr>
<tr>
<td>$K_{at}$ (N-mm)</td>
<td>967440</td>
</tr>
<tr>
<td>$K_{ab}$ (N-mm)</td>
<td>0</td>
</tr>
<tr>
<td>$K_{ta}$ (N-mm)</td>
<td>967440</td>
</tr>
<tr>
<td>$K_{tt}$ (N-mm$^2$)</td>
<td>844462</td>
</tr>
<tr>
<td>$K_{tb}$ (N-mm$^2$)</td>
<td>0</td>
</tr>
<tr>
<td>$K_{bt}$ (N-mm)</td>
<td>0</td>
</tr>
<tr>
<td>$K_{bb}$ (N-mm$^2$)</td>
<td>2375885</td>
</tr>
</tbody>
</table>

Comparing the stiffness coefficients of constrained bending over the present free bending case, the analytical results are presented in Table 6.2. the flexural rigidity coefficient showed a significant variation of 24% while others recorded less than 15% respectively. This is attributed to the inclusion of the effects of the interaction between the drum and the strand. The validation was done with finite element model developed.

The Figure 6.5 shows the bending stress distribution of the stranded cable for the transverse load applied through the drum sector. A pressure load was applied on the drum in small increments and the effect on the cable was recorded. As the pressure is increment the radius of curvature and corresponding parameters also vary. This has been extracted for various load
steps. One such increment of pressure load applied on the drum during the analysis, has resulted in 4.8 m radius of curvature of cable and the corresponding bending stress pattern for the strand as shown in Figure 6.5

**Figure 6.5** Bending stress on a strand for radius of curvature of 60mm induced. (a) stress plot across the span, (b) stress plot at the area of interaction between the drum and cable
Figure 6.6  Contact stress developed at the wire interactions. (a) Contact stress plot across the span (b) Contact stress plot at the area of interaction between the drum and cable
The stress peaks are located very near the contact surfaces and the highest stress occurs at the contact between the bottom most helical wire and the drum. This bottom most wire experiences additional wire forces and moments from the drum which result in increased stiffness when compared to other wires of the layer. Also, the global strand stiffness is also affected as shown in Table 6.2 where been compared stiffness for free and constrained bending of the present model have.

The Figure 6.7 shows the dependence of bending moment for various radii of curvature of loading. The present theoretical model agrees well with the findings from the finite element analysis of the strand. A neglecting low value of Poisson’s ratio of 0.001 was adopted in the finite element analysis only for the purpose of solving.

![Figure 6.7 Variation of Bending Moment as a function of radius of curvature](image)

Figure 6.7 Variation of Bending Moment as a function of radius of curvature

The present analytical model does not include the Poisson’s effect and hence a comparison for the variation of bending moment versus radius of
curvature was attempted. The findings from the finite element analysis underestimated the theoretical model by a value 8.6 % for the range of radius of curvature presented in Figure 6.7.

6.6 CONCLUSION

This chapter presents the evaluation of effective strand axial, torsional and bending stiffness for single layered constrained bending case and the results have been compared with that of free bending case of present model. This approach has been used, as there has been no mathematical model addressed in the literature on the response of the stranded cable under constrained bending. The study of the variation in the stiffness coefficients for the both free bending case and constrained bending cases was performed. Further the analytical model was compared with the finite element analysis and the findings have been reported.