Chapter 4

A Study of the Vorticity Balance in the Tropical Upper Troposphere

4.1 Introduction

A simple scale analysis, of the large scale motions in the tropical atmosphere, sug­gests that the primary thermodynamical balance occurs between the diabatic heating (cooling) term and the adiabatic cooling (warming) due to vertical ascent(descent). This essentially means that, in the tropics, the thermodynamic energy equation can be approximated as a diagnostic balance relation. Similarly, if one considers the tropical atmospheric motions having large spatial and temporal scales, it can be shown that the divergence equation also reduces to a balance condition. We are therefore effectively left with a single prognostic relation (i.e., the vorticity equation) which contains vital information regarding the spatial and temporal evolutionary characteristics of the large scale circulations in the tropical atmosphere. Hence, a diagnostic analysis of the various terms in the vorticity equation can be used as a powerful tool to understand the dynamics of the atmospheric motions in the tropics. The relevance of this study is to examine the nature of vorticity balance that governs the time-mean
circulations in the tropical atmosphere.

4.1.1 Observational and diagnostic studies

Let us begin our discussion with some of the important observational features associated with large scale motions in tropics. Krishnamurti (1971) and Krishnamurti et al. (1973) have given a comprehensive account of the planetary scale east-west circulations and quasi-stationary waves, during the northern summer and winter months. The salient features of the upper tropospheric circulations during the northern summer, are the Tibetan and Mexican anticyclones, the easterly jet over south east Asia, the mid-oceanic troughs over the Atlantic and Pacific and finally the southern hemispheric westerly subtropical jet. Krishnamurti (1971) found that the thermally direct east-west overturnings, caused by land-ocean heating and zonal asymmetries, were more pronounced in the northern summer than during other seasons. It was found that the convective latent heat release, which was the major component of heating, had maximum intensity over the Tibetan and Mexican highlands. The zonal temperature contrast at the upper levels between the warm southwest monsoon circulation over Asia and the oceanic trough over the Pacific was shown to be as large as 25°C. Krishnamurti (1971) found that the intensity of the east-west circulations was comparable to that of the Hadley type circulation. During winter, intense convective heating occurs over the maritime continent of Malaysia-Indonesia resulting in upper tropospheric anticyclones over south China sea and western Pacific. In addition, two weak highs appear over equatorial eastern Africa and the northern part of South America. Oceanic troughs can be seen extending from the equator to the southern subtropics. The most outstanding feature during the winter as well as the summer months is an almost exact out of phase relation between the observed vorticity and divergence fields in the upper troposphere.
Many observational and diagnostic studies have examined the vorticity dynamics in the tropical atmosphere and estimated the significance of vertical transport of vorticity by cumulus convection. Williams and Gray (1973), Reed and Johnson (1974), Hodur and Fein (1977) and Chu et al. (1981) have performed vorticity budget analyses over the tropical western Pacific. Reed and Johnson (1974) found that there existed an apparent vorticity sink for the large-scale motions in the lower half of the troposphere and an apparent vorticity source in the upper troposphere. They attributed this to the removal of vorticity-rich air from the lower layers and its deposition at the upper layers by deep convection. Hodur and Fein (1977) found that subgrid scale cumulus convection processes played an important role in the vertical transport of vorticity in the tropical oceanic region. Similar diagnostic studies over the eastern Atlantic ocean have been carried out, using the data from GARP Atlantic Tropical Experiment (GATE), by Shapiro (1978), Stevens (1979) and Reeves et al. (1979). Shapiro (1978) and Stevens (1979) studied the synoptic scale wave disturbances over Africa and eastern Atlantic and found that the disturbances were governed by linear dynamics. Reeves et al. (1979) examined the interrelationship between the convective scale precipitation and the large scale wind field using the upper air and surface data from the GATE. They concluded that vorticity was transported from the surface to the top of the atmosphere due to cumulus convection. In a similar study, Sui and Yanai (1986) have shown that organized cumulus convection affects the large scale vorticity budget, resulting in the deceleration of upper tropospheric flow. Fein (1977) evaluated the large scale terms in the vorticity equation using the summer mean fields at 200 mb from Krishnamurti's (1971) analysis. His study showed that there is a requirement for a subgrid scale mechanism which rapidly removes negative vorticity from the regions of strong divergence (Tibetan and Mexican highlands) and removes positive vorticity from the regions of strong convergence (mid-oceanic troughs) at 200 mb. He concluded that over regions of strong and persistent convection such as
the Tibetan plateau, deep cumulus clouds can account for the transport. However, the mechanism for removing positive vorticity in the vicinity of upper tropospheric mid-oceanic troughs was still not clearly understood.

4.1.2 Theoretical studies based on linearized models

Holton and Colton (1972) employed a linearized barotropic steady state vorticity equation to study the vorticity balance at the upper troposphere during JJA 1967. The barotropic vorticity equation, linearized about the zonal mean wind $[\bar{u}]$, is given below. The zonal mean is denoted by $[\bar{\cdot}]$ and the deviation from the zonal mean by $\bar{\cdot}$. Similarly, an overbar denotes the time-mean and a prime denotes the deviation from the time-mean.

$$\frac{[\bar{u}]}{a \cos \phi} \frac{\partial}{\partial \lambda} \nabla^2 \psi^* + \frac{1}{a \cos \phi} \frac{\partial \psi^*}{\partial \lambda} \left( \beta + \frac{1}{a} \frac{\partial \bar{c}}{\partial \phi} \right) = - (f + \bar{\zeta}) \nabla . \vec{V}$$

$$-\varepsilon \nabla^2 \psi^* \quad (4.1)$$

They linearized the vorticity equation about a zonal mean flow at 200 mb and forced it by the observed time-mean horizontal divergence. In (4.1), $-(f + \bar{\zeta}) \nabla . \vec{V}$ is the time-independent forcing at 200 mb. The zonal mean flow at 200 mb ($[\bar{u}]$and $[\bar{\zeta}]$) and the time-mean horizontal divergence for JJA 1967 were obtained from Krishnamurti's (1971) analysis. They calculated the response to the observed forcing, by solving (4.1) for different values of $\varepsilon$. It was found that the calculated streamfunction resembled the observed streamfunction only for the values of $\varepsilon$ corresponding to damping times of less than one day. However when a weak vorticity dissipation was used, the mismatch between the calculated and observed streamfunction distribution at 200 mb was large. It can be seen from Figs.4.1a and 4.1b that, in the presence of a strong viscous damping of vorticity ($1.5 \times 10^{-5} S^{-1}$), the structure and intensity of the upper level easterlies and the Tibetan anticyclone closely match with the observed flow field.
Fig. 4.1a. (Top) Observed 200 mb perturbation streamfunction during summer 1967. (Adapted from Holton and Colton (1972)). Interval $5 \times 10^6 \, \text{m}^2 \, \text{s}^{-1}$.

Fig. 4.1b. (Bottom) Computed perturbation streamfunction using linear barotropic vorticity model, for damping rate of $1.5 \times 10^{-5} \, \text{s}^{-1}$. (Adapted from Holton and Colton (1972)). Interval $5 \times 10^6 \, \text{m}^2 \, \text{s}^{-1}$. 
It was proposed that the vorticity damping at the upper layers could arise because of vertical transport of horizontal momentum due to deep cumulus convection. They conjectured that the deep cumulonimbus towers in the tropics could transport vorticity directly from the boundary layer to the outflow level at 200 mb within a time scale of one day. The vorticity is subsequently deposited at 200 mb which later on decays quite rapidly.

4.1.3 Theoretical studies based on nonlinear models

The significance of nonlinear dynamics in the tropical upper troposphere has been pointed out by several investigators. Colton (1973) examined the nonlinear interactions between quasi-stationary long waves and transient synoptic waves in the tropical upper troposphere. He showed that a nonlinear barotropic energy exchange between large scale quasi-stationary forced waves and transient synoptic scale waves was important in producing several features of the upper tropospheric general circulation (for example, the vortices and small scale waves present in the mid-oceanic troughs and the disturbances that form along the easterly jet over the Indian ocean). He also found that the nonlinear transfer of energy to smaller scales of motion acted as a damping mechanism to the large scale forced circulation. Chang (1977) has shown that the barotropic instability associated with the planetary scale summer monsoon leads to a strong nonlinear energy transfer which appears to be quite relevant to the damping of planetary scale flows during the summer monsoon. Kanamitsu (1977) performed model studies and obtained a reasonable position of the Tibetan high even without the inclusion of strong damping. His vorticity budget study at 200 mb suggested that the advection of relative vorticity by the divergent part of the flow is one of the leading terms of the vorticity equation. His study indicated that the tropical quasi-stationary ultralong waves are fully nonlinear and nongeostrophic during the
Indian Summer Monsoon. Cho et al (1983) studied the vorticity dynamics of tropical easterly waves obtained in GATE using a simplified vorticity equation and found that horizontal advection of vorticity by the African easterly jet considerably influenced the evolution of the vorticity field. Sardeshmukh and Held (1984) studied the vorticity budget in the upper tropospheric levels of the tropical atmosphere based on GCM simulations of the northern summer monsoon performed at Geophysical Fluid Dynamics Laboratory (GFDL). The model did not have explicit momentum transfer in the cumulus parameterization. The model could still simulate the 200 mb circulation and the Tibetan anticyclone quite realistically. However, on forcing the linearized barotropic vorticity equation using the model’s zonal mean flow and mean divergence, the Tibetan anticyclone could be reproduced only when large values of the damping coefficient were used. This damping was found to crudely account for the neglected nonlinear horizontal advection. Their study showed that nonlinear interactions in the tropics could possibly play the role of vorticity damping.

In a subsequent study, Sardeshmukh and Hoskins (1985) carried out detailed budget analysis of the vorticity balance at 150 mb in the tropics during December 1982 to February 1983. This season was characterized by warm SST anomalies over the central and east equatorial Pacific ocean. There was a shift of the maximum tropical rainfall and cloudiness from Indonesia - New Guinea to the central Pacific. The observations of 1982-83 El Niño-Southern Oscillation (ENSO) by Quiroz (1983), Rasmusson and Wallace (1983) clearly indicate anomalous low level westerlies to the west of the heating anomaly. Anticyclones were seen straddling the equator at the same longitude as the heating anomaly. They found that the observed anomalies at 150 mb could not be modelled within the frame work of a steady state model linearized about a basic flow at rest, where the vorticity equation takes the form:

$$ \beta v = -f \nabla \cdot \vec{V} - \epsilon \zeta $$  \hspace{1cm} (4.2)
where $f = \beta y$. For small $\varepsilon$, the vorticity source $f\frac{\partial \omega}{\partial p}$ is balanced by $\beta v$ with a result that the high at the upper level always occurs to the west of the heating region. But observations (e.g., Krishnamurti (1971)) suggest that the upper level anticyclone is almost in phase with the heating distribution. Therefore linear models have no option but to employ a strong vorticity damping, as a result of which, a balance between the vorticity source term and the friction term occurs, thus overcoming the phase-shift problem. The strong damping, used in the linear models, is often associated with the vertical transport of vorticity in cumulus towers. It should however be noted that the GCM simulations of Blackmon et al. (1983), Sardeshmukh and Held (1984) and the nonlinear model calculation of Hendon (1986) have overcome the phase-shift problem inspite of not incorporating vertical momentum transport in the cumulus convection parameterization. Using a nonlinear model, Hendon (1986) calculated the steady state response due to an isolated heat source symmetric w.r.t the equator. He showed that when the forcing was quite large, the nonlinear advection terms displaced the upper level anticyclones eastwards to the same longitude as the heating. The vorticity budget calculations of Sardeshmukh and Hoskins (1985) revealed that the dominant vorticity balance in the tropics was between the stretching and advection of absolute vorticity by the time-mean horizontal flow. In another study, Sardeshmukh and Hoskins (1988) have shown that the advection of vorticity by the divergent component of the flow is very important in determining the response to tropical heating.

4.2 Motivation for the present study

From the above mentioned studies of the vorticity balance in the tropics, we have seen that there are differences between the linear and nonlinear calculations. The
linear calculations assume a large vorticity damping, which is associated with vertical transport of momentum by cumulus convection, in order to explain the observed circulation features. In other words, the linear models assume that deep cumulus clouds rapidly transport vorticity from the boundary layer to the top of the atmosphere in < 1 day. However, such a rapid decay of the upper tropospheric vorticity may not be quite physical. On the contrary, the nonlinear calculations indicate that nonlinearities play a more fundamental role in determining the observed time-mean vorticity balance in the tropics. The nonlinear studies also indicate that the vertical transport of momentum by cumulus convection has only secondary effects on the time-mean large scale circulation in the tropics.

In Chapter II, we had studied the impact of nonlinear terms on the quasi stationary waves in the tropical atmosphere. It was shown that nonlinearities can significantly modify the structure of the time-mean response in the vicinity of a strong convective heat source. One can actually estimate the nonlinear effects in the tropical atmosphere, by performing diagnostic calculations of the vorticity equation. In this chapter, we shall carry out vorticity budget calculations, using the output fields generated from steady state forcing experiments with a 5-level nonlinear global spectral model. Our intention is to understand the nature of vorticity balance in the tropical upper troposphere. The analysis will be carried out for the data sets generated by three different forcing experiments with the 5-level nonlinear global spectral model.

- Model forced with an idealized heating distribution.
- Model forced with the diabatic heating for DJF-(1978-79).
- Model forced with the diabatic heating for JJA-(1979).

Sardeshmukh and Hoskins (1985) assumed that the vertical advection and twisting terms are quite small in the tropics. In our study we have made an assessment of the
various terms in the vorticity equation, including the vertical advection and twisting terms, and their contribution towards the vorticity balance in the tropical upper troposphere.

4.3 The terms in the vorticity equation

Let us now consider the vorticity equation in pressure coordinates. The external sources and sinks and subgrid-scale processes are ignored in the equation.

\[
\frac{\partial \zeta}{\partial t} + \vec{V} \cdot \nabla (\zeta + f) + \hat{k} \cdot \nabla \times \left( \frac{\partial \vec{V}}{\partial p} \right) = - (\zeta + f) \nabla \cdot \vec{V} \quad (4.3)
\]

In (4.3) \( \omega \) is the vertical p velocity, \( \vec{V} \cdot \nabla (\zeta + f) \) represents the advection of absolute vorticity by the horizontal wind and \( \hat{k} \cdot \nabla \times \left( \frac{\partial \vec{V}}{\partial p} \right) \) is the vertical advection and twisting term. The term \( -(\zeta + f) \nabla \cdot \vec{V} \) represents the stretching term. Equation (4.3) is now averaged over a period of time. The vorticity balance reduces to

\[
\bar{V} \cdot \nabla (\zeta + f) + \bar{k} \cdot \nabla \times \left( \frac{\partial \vec{V}}{\partial p} \right) = - (\zeta + f) \nabla \cdot \bar{V} \quad (4.4)
\]

One can rewrite (4.4) as:

\[
\bar{V} \cdot \nabla (\zeta + f) + \bar{V'} \cdot \nabla \zeta' + \bar{k} \cdot \nabla \times \left( \frac{\partial \vec{V}}{\partial p} \right) + \bar{k} \cdot \nabla \times \left( \frac{\partial \vec{V'}}{\partial p} \right)
= - (\zeta + f) \nabla \cdot \bar{V} - \zeta' \nabla \cdot \bar{V'} \quad (4.5)
\]

For convenience we shall refer the terms involving the time-mean namely, \( \bar{V} \cdot \nabla \)
\[ (\zeta + f) \] as M1, \(- (\zeta + f) \nabla \cdot \vec{V} \) as M2 and \(\hat{k} \nabla \times \left( \omega \frac{\partial \vec{V}}{\partial p} \right) \) as M3. The terms involving the transients namely, \(\vec{V}' \nabla \zeta' \) as T1, \(- \zeta' \nabla \vec{V}' \) as T2, and \(\hat{k} \nabla \times \left( \omega' \frac{\partial \vec{V}'}{\partial p} \right) \) as T3.

### 4.4 Vorticity balance at 300 mb in a global spectral model

We first consider the case of an idealized forcing which essentially is a combination of a heat source and heat sink symmetric w.r.t the equator (Fig.4.2a). The centres of the heat source and the heat sink are separated by 112.5° longitudes. This form of forcing very crudely mimics the heating in the western Pacific and the longwave cooling in the eastern Pacific. The vertical distribution of the forcing is of the form \( \sin \left( \frac{\pi p}{p_0} \right) \) which has a maximum value at 500 mb and a minimum at the surface and top of the atmosphere. Using the above forcing the model was integrated for a period of 70 days until a steady state was reached. A weak Rayleigh friction having a dissipation time scale of 15 days was used in the model. The stationary Kelvin and Rossby waves induced by this form of forcing have already been discussed thoroughly in Chapter II. The region of the Walker circulation between the heat source and the heat sink is characterized by upper level westerlies and lower level easterlies. There is rising motion over the heating region and subsidence over the heat-sink. The steady-state streamfunction and velocity potential at 300 mb are shown in Figs.4.2b and 4.2c respectively. It can be seen from Fig.4.2b, that in the region between the source and the sink, the streamfunction at 300 mb has the structure of a dipole in the meridional direction. From the velocity potential distribution at 300 mb, one can infer the regions of divergence and convergence at the upper level. The negative (positive) contours of velocity potential correspond to regions of outflow (inflow). It is evident
Fig. 4.2a. (Top) Latitude-longitude section of vertically averaged heating (idealized case). Interval 0.2 °K per day.

Fig. 4.2b. (Middle) Computed streamfunction at 300 mb using 5-level nonlinear global spectral model. Forcing in Fig. 4.2a used. Interval $1\times10^5 \text{m}^2\text{s}^{-1}$.

Fig. 4.2c. (Lower) Same as Fig. 4.2b, except for velocity potential. Interval $1\times10^6 \text{m}^2\text{s}^{-1}$. 
from Fig.4.2c that at the upper levels, there is strong divergence in the region of the heat source and a converging flow over the heat sink. The meridional gradient of the streamfunction, in between the heat source and heat sink, is negative. In addition, the longitudinal gradient of the velocity potential is positive. The meridional gradient of the streamfunction and the zonal gradient of the velocity potential, in the region between the heat source and the heat sink, consistently support the westerlies at 300 mb.

The time mean and transient terms of the vorticity equation are constructed using the fields from day 50 to day 70. The different terms of the equation 4.5 are shown in Figs.4.3a-4.3f. Area average values for these terms are shown in Table.4.1. These averages are calculated for the domain: 67.5°E to 157°E in the zonal direction and 3.4°N to 44.1°N in the meridional direction, which represents the region between the heat source and the heat sink. It can be clearly seen from the area average values that the advection of absolute vorticity by the time-mean horizontal wind (M1 = 5.3x10^{-12} S^{-2}) and the vorticity stretching term due to the time-mean wind (M2 = 4.4x10^{-12} S^{-2}) are the dominant terms in the vorticity equation. In addition it can be noted from Fig.4.3a and Fig.4.3b that the structure and magnitude of the terms M1 and M2 nearly match. This indicates that the predominant vorticity balance, in the tropical upper troposphere, is between the horizontal advection of absolute vorticity and the vorticity stretching term. It should be recognized that this balance has been achieved in the presence of a very weak Rayleigh friction in the model. The advection of absolute vorticity by horizontal wind (M1) and the absolute vorticity stretching by the horizontal divergence (M2) are basically nonlinear terms in the vorticity equation. Therefore, a primary balance between the terms M1 and M2 as seen from Figs.4.3a and 4.3b, highlights the significance of nonlinear dynamics in the tropical upper troposphere. It is further seen from Fig.4.3c that the term representing the time-mean vertical advection and twisting of vorticity (M3) is three
Fig. 4.3a. (Top) Latitude-longitude section of Term M1. Case of Idealized forcing. Interval $0.6 \times 10^{-11} s^{-2}$.

Fig. 4.3b. (Middle) Same as Fig. 4.3a, except for term M2. Interval $0.64 \times 10^{-11} s^{-2}$.

Fig. 4.3c. (Lower) Same as Fig. 4.3a, except for term M3. Interval $0.1 \times 10^{-11} s^{-2}$.
Fig. 4.3d. (Top) Same as Fig. 4.3a, except for term T1. Interval $0.044 \times 10^{-14} s^{-2}$.
Fig. 4.3e. (Middle) Same as Fig. 4.3a, except for term T2. Interval $0.044 \times 10^{-14} s^{-2}$.
Fig. 4.3f. (Lower) Same as Fig. 4.3a, except for term T3. Interval $0.006 \times 10^{-14} s^{-2}$.
### Table 4.1: Area Averages of the Terms of the Vorticity Equation

<table>
<thead>
<tr>
<th>FORCING</th>
<th>AVERAGING REGION</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idealized Source-Sink</td>
<td>67.5°E-157.5°E 3.4°N - 44.1°N</td>
<td>5.3x10^{-12}s^{-2}</td>
<td>4.4x10^{-12}s^{-2}</td>
<td>3.0x10^{-16}s^{-2}</td>
<td>4.6x10^{-17}s^{-2}</td>
<td>1.14x10^{-17}s^{-2}</td>
<td>4.4x10^{-15}s^{-2}</td>
</tr>
<tr>
<td>combination</td>
<td>DJF-1979 101.25°E-101.25°W -15.5°S - 28.8°N</td>
<td>2.3x10^{-11}s^{-2}</td>
<td>2.4x10^{-11}s^{-2}</td>
<td>1.08x10^{-12}s^{-2}</td>
<td>1.9x10^{-14}s^{-2}</td>
<td>1.03x10^{-14}s^{-2}</td>
<td>3.8x10^{-15}s^{-2}</td>
</tr>
<tr>
<td></td>
<td>JJA-1979 45°E-123.75°W -11.1°S - 33.3°N</td>
<td>2.6x10^{-11}s^{-2}</td>
<td>1.8x10^{-11}s^{-2}</td>
<td>1.13x10^{-12}s^{-2}</td>
<td>1.18x10^{-12}s^{-2}</td>
<td>1.5x10^{-12}s^{-2}</td>
<td>9.4x10^{-16}s^{-2}</td>
</tr>
</tbody>
</table>
to four orders smaller than the terms M1 and M2. The smallness of M3 is also seen in its area average value, in the region between is heat source and sink, which is $3 \times 10^{-17} \text{S}^{-2}$. Terms involving the transients T1, T2 and T3 are given in Figs. 4.3d, 4.3e and 4.3f respectively. It is clearly evident that the terms T1, T2 and T3 and also their respective area averages are negligible as compared to the terms M1 and M2.

A similar vorticity budget study has been carried out for the winter 1978-79 dataset. The 5-level nonlinear global spectral model was forced using the observed time-mean 3-dimensional atmospheric heating for DJF (1978-79). The observed diabatic heating distribution was prescribed at all the model levels. The vertically averaged heating is shown in Fig. 4.4a. The large scale features consist of the heating over Indonesia and western Pacific, continental heating over South America and Africa. The time-mean response induced by the observed diabatic heating for winter (1978-79) was calculated in the 5-level nonlinear global spectral model. In the last chapter, we had given a detailed account of the planetary scale circulations during DJF (1978-79). The model generated streamfunction and velocity potential at 300 mb are shown in Fig. 4.4b and Fig. 4.4c respectively. The most prominent planetary scale circulation during the northern winter is the Walker circulation in the equatorial Pacific. Large scale divergence over Indonesia and subsidence over the eastern Pacific can be seen in Fig. 4.4c. The equatorial Pacific is characterized by upper level westerlies and lower level easterlies. The upper level westerly flow in the equatorial Pacific is evident from the positive values of the zonal gradient associated with velocity potential at 300 mb.

The different terms of the equation 4.5 for the winter 1979 analysis are shown in Figs. 4.5a-4.5f. The area average values for the different terms are calculated in the region between 101.25°E and 101.25° W and -15.5° S and 28.8° N, which corresponds to the Walker circulation in the equatorial Pacific. It can be seen from the area
Fig. 4.4a. (Top) Same as Fig. 4.2a, except for the case of winter (1978-79). Interval 0.58 °K per day.

Fig. 4.4b. (Middle) Same as Fig. 4.2b, except for the case of winter (1978-79). Interval 30x10^5 m^2 s^-1.

Fig. 4.4c. (Lower) Same as Fig. 4.2c, except for the case of winter (1978-79). Interval 15x10^5 m^2 s^-1.
Fig. 4.5a. (Top) Same as Fig. 4.3a, except for the case of winter (1978-79). Interval 5x10^{-11}s^{-2}.

Fig. 4.5b. (Middle) Same as Fig. 4.3b, except for the case of winter (1978-79). Interval 4x10^{-11}s^{-2}.

Fig. 4.5c. (Lower) Same as Fig. 4.3c, except for the case of winter (1978-79). Interval 0.4x10^{-11}s^{-2}.
Fig. 4.5d. (Top) Same as Fig. 4.3d, except for the case of winter (1978-79). Interval $0.2 \times 10^{-11} \text{s}^{-2}$.

Fig. 4.5e. (Middle) Same as Fig. 4.3e, except for the case of winter (1978-79). Interval $0.055 \times 10^{-11} \text{s}^{-2}$.

Fig. 4.5f. (Lower) Same as Fig. 4.3f, except for the case of winter (1978-79). Interval $0.012 \times 10^{-11} \text{s}^{-2}$.
average values and from Figs. 4.5a and 4.5b that the terms involving the advection of absolute vorticity by the time-mean horizontal wind and the time-mean absolute vorticity stretching are of comparable magnitudes. The horizontal structures of the terms $M_1$ and $M_2$ are quite similar. The area average values of $M_1$ and $M_2$ are $2.3 \times 10^{-11} S^{-2}$ and $2.4 \times 10^{-11} S^{-2}$ respectively. The time-mean vertical advection and twisting terms shown in Fig. 4.5c are about one to two orders smaller than the terms $M_1$ and $M_2$. The terms involving the transients $T_1$, $T_2$ and $T_3$ (Figs. 4.5d, 4.5e and 4.5f) and also their area average values are negligibly small as compared to the terms time-mean terms $M_1$ and $M_2$. The terms $M_1$ and $M_2$ being the dominant terms during winter (1978-79), suggest that nonlinear effects associated with the advection and stretching of absolute vorticity by the horizontal winds are quite important in determining the vorticity balance over the region of the Walker circulation in the equatorial Pacific.

We shall now examine the vorticity balance associated with the summer (1979) monsoon circulation. Fig. 4.6a shows the vertically integrated time-mean heating during JJA (1979). The large heating over the Bay of Bengal, the foothills of Himalayas and the elevated Tibetan plateau, drives the summer monsoon circulation over India. The mean meridional circulations as well as the major east-west circulations forced by the time averaged heating during summer (1979) were extensively discussed in the previous chapter. The dataset for the vorticity budget study was obtained by forcing the 5-level nonlinear global spectral model with the observed three dimensional time-mean diabatic heating for JJA (1979). A weak Rayleigh friction of 15 days dissipation time scale was used in the model. The model was integrated for 100 days so as to attain a steady state. The steady-state streamfunction and velocity potential at 300 mb are shown in Fig. 4.6b and Fig. 4.6c respectively. The most striking feature in the upper levels is the anticyclonic flow over the Tibetan region. In addition to the Hadley circulation of the Indian summer monsoon, there are prominent planetary
Fig. 4.6a. (Top) Same as Fig. 4.2a, except for the case of summer (1979). Interval 0.5 °K per day.
Fig. 4.6b. (Middle) Same as Fig. 4.2b, except for the case of summer (1979). Interval $80 \times 10^8 m^2 s^{-1}$.
Fig. 4.6c. (Lower) Same as Fig. 4.2c, except for the case of summer (1979). Interval $25 \times 10^8 m^2 s^{-1}$.
scale east-west circulations associated with the heating in the Tibetan and Mexican highlands. The velocity potential at 300 mb shows outflow over the Indian summer monsoon region and the highlands of Mexico.

The time mean and transient terms of the vorticity equation were constructed using the fields of the last 20 days. The different terms of the equation 4.5 for the summer 1979 analysis are shown in Figs.4.7a-4.7f. It is seen from Figs.4.7a and 4.7b, that the magnitude and structure of the time-mean terms associated with horizontal advection and stretching of absolute vorticity, match well especially over the highlands of Tibet and northern central Pacific. The time-mean vertical advection and twisting terms are about one order smaller than M1 and M2 over the Indian monsoon region. The transient terms T1, T2 and T3, shown in Figs.4.7d, 4.7e and 4.7f are small in the low latitudes. The magnitude of the transient term T1 over the Tibetan plateau is nearly the same as the time-mean term M3. The area average values for the different terms have been calculated in the region between 45° E and 123.75° E and 11.5° S and 33.3° N, which is the region of the northern summer monsoon. The area average values of M1, M2 and M3 are $2.6 \times 10^{-11} \, \text{s}^{-2}$, $1.8 \times 10^{-11} \, \text{s}^{-2}$ and $1.13 \times 10^{-12} \, \text{s}^{-2}$ respectively. It can be observed that the area average values of M1 and M2 are of comparable magnitudes while M3 is one order smaller than M1 and M2. The area average values of transient terms T1 and T2 are 1-2 orders smaller than the the corresponding time-mean terms.

4.5 Concluding remarks

We have performed a diagnostic study of the vorticity balance in the tropical upper troposphere using the datasets generated from steady-state forcing experiments with
Fig. 4.7a. (Top) Same as Fig. 4.3a, except for the case of summer (1979). Interval $9.4 \times 10^{-11} \text{s}^{-2}$.

Fig. 4.7b. (Middle) Same as Fig. 4.3b, except for the case of summer (1979). Interval $9 \times 10^{-11} \text{s}^{-2}$.

Fig. 4.7c. (Lower) Same as Fig. 4.3c, except for the case of summer (1979). Interval $1 \times 10^{-11} \text{s}^{-2}$. 
Fig. 4.7d. (Top) Same as Fig. 4.3d, except for the case of summer (1979). Interval $2 \times 10^{-11} \text{s}^{-2}$.

Fig. 4.7e. (Middle) Same as Fig. 4.3e, except for the case of summer (1979). Interval $1 \times 10^{-11} \text{s}^{-2}$.

Fig. 4.7f. (Lower) Same as Fig. 4.3f, except for the case of summer (1979). Interval $0.23 \times 10^{-11} \text{s}^{-2}$. 
a 5-level nonlinear global spectral model. The purpose of this study was to determine the nature of the time-mean vorticity balance in the tropical upper troposphere and also estimate the various terms in the vorticity equation. We have utilized three datasets for our study which are based on the model outputs of three different forcing experiments. An idealized heat source-heat sink combination was used as the forcing in the first case. In the second case the model was forced using the observed diabatic heating for winter 1978-79 and in the third experiment the forcing was based on the observed diabatic heating during summer 1979. For the above three cases the model was integrated and the steady state response was determined. In our forcing experiments we had employed very weak damping terms in the model. Using the model generated fields the various terms in the vorticity equation at 300 mb were diagnosed and their area averages were computed. For the case of the idealized experiment, the region between the heat source and the sink was chosen for calculating the area averages. For the case of winter 1979, the area averages were determined in the region of the Walker circulation over the equatorial Pacific. In the case of summer 1979, the averages were calculated for the region covering the northern summer monsoon area.

The key result of our diagnostic study shows that the primary vorticity balance in the tropical upper troposphere is between the vorticity stretching and the advection of absolute vorticity by the time-mean horizontal winds. This result agrees with the findings of Sardeshmukh and Hoskins (1985). Our budget study in all the three cases shows that these two terms are larger than the time-mean term involving the vertical advection and stretching of vorticity. The transient terms in the vorticity equation were found to be generally very small in the low latitudes. However, in the case of JJA (1979), the transient term $T_1$ was only about 1 order smaller than the time-mean term $M_1$ over the Indian summer monsoon region. It should be noted that the terms representing horizontal advection and stretching of absolute vorticity, are basically nonlinear terms in the vorticity equation. From the largeness of these two terms
in the tropics, it can be concluded that nonlinear effects associated with horizontal advection and vorticity stretching, can greatly affect the large-scale motions in the tropical upper troposphere.