Chapter 2

Dynamics of the Time-Mean Walker Circulation

2.1 Introduction

The most spectacular observational feature in the tropical atmosphere is the strong longitudinal variation in the distribution of heat sources and heat sinks (Ramage (1968), Krueger and Winston (1974), Heddinghaus and Krueger (1981)). This zonally asymmetric distribution of heating drives the planetary scale circulations in the tropical troposphere. The tropical atmosphere is also characterized by meridional circulations which are induced by north-south heating contrasts. A study of the heat induced stationary response of the tropical atmosphere provides valuable information about the dynamics of the time-mean large scale motions in the tropics. In addition to the time-mean motions, the tropical atmosphere also exhibits variability on different time-scales. The year to year variability of the Indian summer monsoon circulation, the Walker circulation in the equatorial Pacific and the phenomenon of El Niño and Southern Oscillation (ENSO) are some of the challenging problems pertaining to the interannual variability in the tropics. Climate transitions on the seasonal time-scale are marked by considerable meridional shifts in the equatorial convective activity.
There is a major need to understand the structure and dynamics of the circulations that are induced by these seasonal shifts in the diabatic heating. On the intraseasonal time-scale, the dominant variability occurs with a periodicity of about 30-50 days in the tropical atmosphere. The tropical low-frequency 30-50 day oscillation manifests itself as a slow eastward propagation of equatorial convective activity. Transients on even smaller time-scale of a few days occur in association with tropical depressions and cyclones. It is the wide range of spatial and temporal scales comprising the tropical atmospheric motions, that renders it an interesting subject. In this chapter, we shall be primarily focussing on the behavior of the time-mean Walker circulation in the equatorial Pacific. Before starting off with our actual problem, we shall turn to some of the relevant details, which will be described in the following subsections.

2.1.1 The diabatic heating in the tropical atmosphere

The diabatic heating in the tropics, which is largely condensational heating due to cumulus convection, is an important component of the tropical atmospheric system. The annual mean diabatic heating, based on ECMWF analyses for 1979-1989, by Hoskins \textit{et al} (1989) reveals that deep convective heating occurs predominantly over three zones in the tropics located over equatorial South America, equatorial Africa and the 'maritime' continent of Indonesia. These three primary heating zones are responsible for the major east-west circulations in the tropical troposphere. There are also dry and cloud free regions in the tropics, where the radiative cooling in the atmosphere is substantial. For instance the region over central-eastern Pacific, which is characterized by widespread subsidence and strong surface easterly trades, shows considerable longwave cooling. One of the most striking divergent flow patterns in the tropical atmosphere is the time-mean Walker circulation in the equatorial Pacific. The heat budget study by Cornejo-Garrido and Stone (1977) suggest that zonal variation of condensational heating, caused by moisture convergence, primarily
forces the Walker circulation. The strong upward motions over Indonesia and western Pacific result in upper level divergence and east-west circulations. The eastward branch descends in the eastern Pacific and the westward branch descends near Saudi Arabia. The scale of the eastward branch is typically of the order of 15000-20000 kms. Some of the important theoretical investigations of the thermally forced planetary scale tropical atmospheric circulations are due to Webster (1972), Gill (1980), Geisler (1981), Simmons (1982), Rosenlof et al (1986), Schneider (1987) and Ting and Held (1990).

The heating distribution in the tropics exhibits variability on different time-scales. Observations by Krueger and Winston (1974) and studies based on Outgoing Longwave Radiation (OLR) by Liebmann and Hartmann (1982) reveal that there is a significant year to year variability in the distribution of heat sources and heat sinks in the tropics. Krueger and Winston (1974) found that the Walker circulation in the equatorial Pacific was very strong during February 1971, while it did not intensify much during February 1969. These contrasting flow patterns during the two years was a consequence of distinct changes in the spatial distribution of tropical atmospheric forcing. It was seen that the heat sink in the equatorial Pacific extended far to the west during February 1971 and on the other hand the dry zone was mostly confined over the eastern south Pacific during February 1969. The findings of Krueger and Winston (1974) convincingly demonstrate that the interannual fluctuations in the geometrical distribution of the heat sources and heat sinks and also their intensities crucially determine the year to year variability in the seasonal-mean planetary scale circulations in the tropics. On the seasonal time scale, there is considerable variability in the distribution of convective heating in the tropics. These changes are marked by distinct seasonal shifts, mostly in the meridional direction, in the locations of the centres of tropical convective activity (Hoskins et al (1989)). For example, the transition from northern winter to northern summer is characterized by a northward shift in
the convective belt from Indonesia to the foothills of the Himalayas. These latitudi-
nal displacements are important in determining the seasonal time-mean circulations.
Coming to the subseasonal time-scale, observations suggest that the dominant low-
frequency variability in the tropical regions occurs with a period of about 30-50 days
and is characterized by a slow eastward propagation of equatorial convective activity.
This low-frequency wave of equatorial convection, is most intense over the Indian
ocean and the western Pacific. One of the interesting phenomena associated with
this low-frequency wave, is its northward migration over India during the northern
summer monsoon.

2.1.2 GCM studies of interannual variability

There are numerous GCM simulations of the tropical atmospheric interannual vari-
ablity, of which we shall mention a few well-known studies. Bjerknes (1969) proposed
that the sea surface temperature (SST) gradients along and south of the equator in the
Pacific are mainly responsible in determining the changes in the Walker circulation
on the interannual time-scale. The study of Horel and Wallace (1981) indicates that
the SST changes in the equatorial Pacific during El Niño years, produce a shift in the
region of major equatorial convective activity. As a result of this displacement, the
maximum precipitation zone moves to the east of Indonesia. Consequently, drought
conditions prevail over tropical Australia, New Guinea and Indonesia. The study
of Horel and Wallace (1981) highlights the significance of tropical SST fluctuations
in producing interannual variations in the tropical atmosphere. Conversely, there
are also studies dealing with the response of the tropical oceans to atmospheric wind
stresses. Wyrtki (1975) demonstrated that El Niño is the result of the response of the
have shown that the surface wind is a key factor that affects the climatological distri-
bution and interannual variability of SSTs in the tropical oceans. It is evident from
the above studies, that there is a strong coupling between the tropical atmosphere and the tropical oceans on the interannual time-scale.

Rasmusson and Carpenter (1982) and Philander (1983) have studied the variations in tropical SST and surface winds associated with the ENSO. Rowntree (1972) used the atmospheric model at Geophysical Fluid Dynamics Laboratory (GFDL) to test Bjerknes' hypothesis that fluctuations of ocean temperatures in the tropical east Pacific are responsible for major variations in the position and intensity of the Aleutian surface low. He found that the SST variations had important effects on the model temperatures both tropical and extra-tropical. More recently, Neelin (1990) and Neelin (1991) have studied the interannual oscillations in the tropical atmosphere, arising out of slow temporal changes in the oceanic surface temperature. Julian and Chervin (1978) performed GCM experiments using the National Center for Atmospheric Research (NCAR) atmospheric model. The boundary forcing was prescribed using the ocean surface temperature. They showed that the coupling between the ocean and the atmosphere was essential for determining the interaction between the two systems. Lau (1981) has shown that coupling between large-scale atmospheric and oceanic equatorial Kelvin waves is relevant in the climatic time scale related to the equatorial ocean/atmosphere processes. Cane et al. (1990) used a linearized version of an analytical model which combines linear ocean dynamics with a simple version of the Bjerknes hypothesis for El Niño. They found that the most important parameter determining the behaviour of the system was the coupling constant. Lindzen and Nigam (1987), Neelin (1989) and Wang and Rui (1990) have demonstrated that the circulation in the atmospheric boundary layer is strongly influenced by the coupling between the boundary layer and the SST. Neelin (1988) used a simple linear model for studying the circulation in the tropical boundary layer. Strong dissipation, associated with vertical mixing of momentum by boundary layer turbulence, was incorporated in the model. The model simulations of the climatology as well as ENSO-type anomalies
agreed with the corresponding results from GCM.

Using a simple model, Webster (1981) showed that when a SST anomaly was located in a low-latitude basic flow, a strong enhancement of the initial anomaly was produced through a positive feedback between the dynamics and diabatic heating. Some of the well-known GCM simulations of ENSO, are due to Keshavamurty (1982), Blackmon et al. (1983) and Fennesy et al. (1985). Keshavamurty (1982) studied the atmospheric response to SST anomalies over the equatorial Pacific using the general circulation model (GCM) of GFDL. He showed that the diabatic heating anomalies associated with the central and western Pacific induce large circulation changes over the equatorial Pacific and south Asia. Philander et al. (1984) showed that unstable interactions between the atmosphere and ocean lead to an amplification of anomalies observed during ENSO. The study by Weare (1986) reveals that changes in evaporation from the sea surface and the surface layer moisture convergence/divergence crucially influence the tropical atmospheric response. Shukla and Fennesy (1988) have examined the tropical rainfall pattern and time-mean circulation for winter 1982 using a global GCM. They found that when observed SST anomalies over the Pacific ocean during the winter of 1982-83 were added to the climatological SST, the model predicted tropical circulation and rainfall were quite realistic.

2.1.3 Stationary and transient waves in the tropical atmosphere

The tropical atmospheric system is characterized by a rich variety of wave motions. It was Matsuno (1966) who first derived the dispersion relation for the wave solutions in the tropical atmosphere using a shallow water system of equations on an equatorial $\beta$-plane. He showed that the normal modes comprise of the equatorially trapped long Kelvin and Rossby waves, and also the high frequency Mixed Rossby-Gravity
long wave approximation by which he eliminated the short wave components. He showed that an isolated heat source symmetric w.r.t the equator triggered Kelvin waves to its east and Rossby waves to its west. A heat source antisymmetric w.r.t the equator produced a Rossby wave response to the west of the heating region. He demonstrated that the Kelvin wave response generated by a symmetric heat source, had a longitudinal extent thrice that of the Rossby wave because of the faster propagation of the Kelvin waves. The model study by Simmons (1982) suggests that the structure of the tropical response to an isolated heat source is sensitive to the vertical profile of heating. He also showed that the magnitude and spatial and temporal variability of the tropical convective heating crucially determine the stationary motion in the tropics. Silva Dias et al. (1983) computed the partition of the thermally forced energy between Kelvin, Mixed Rossby-Gravity, Rossby and Gravity modes. Their model results indicate that many aspects of the summertime upper tropospheric circulation over South America can be explained by the dispersive properties of Rossby and Mixed Rossby-Gravity waves. Kasahara (1984) has studied the forced solutions generated by stationary as well as transient heat sources in the tropics. He found that a stationary heat source could effectively generate Kelvin and Rossby waves, while a transient heat source produced high-frequency gravity waves.

Another interesting aspect pertaining to tropical waves is their interaction with the basic flow. The theoretical studies by Plumb and Bell (1982), Webster and Holton (1982), Wilson and Mak (1984), Kasahara and Da Silva Dias (1986), Webster and Chang (1988), and Chang and Webster (1990) emphasize the significance of the basic flow and also its shear in determining the propagation characteristics and trapping mechanisms for the equatorial waves. Hoskins and Karoly (1981), Lau and Lim (1984) indicated that strong teleconnections from the tropics to midlatitude are possible in a westerly belt over the region of heat source. It can be seen that, during the northern hemispheric winter, the upper troposphere in the mid-Pacific is characterized by a
westerly flow, which enhances the teleconnection between the tropics and extratropics. The studies of Lim and Chang (1983) and Zhang and Webster (1989) indicate that the stationary as well as the transient Rossby wave are less trapped in equatorial westerlies than in an easterly basic state. Lim and Chang (1983), Lim and Chang (1986) and Kasahara and Silva Dias (1986) have pointed out the role of vertical shear in the mean zonal wind in generating external (barotropic) modes which are necessary for teleconnection mechanisms. They have shown that the external modes arise due to interactions between the internal (baroclinic) modes in the presence of vertical shear in the basic zonal wind.

2.2 Aims of the present study

Observations (see Hoskins et. al (1989)) of the annual as well as winter time-mean diabatic heating over the equatorial Pacific, reveals that there is a strong convective heat source over Indonesia and western Pacific and a zone of longwave cooling in the southeastern Pacific. A key question that requires attention, is the dynamics of the time-mean response induced by this combination of heat source and heat sink. One of the problems that is examined in this chapter pertains to the impact of the Indonesian heating and the radiative cooling over the eastern Pacific, in controlling the longitudinal scale and intensity of the time-mean Walker circulation in the equatorial Pacific. The motivation for this work came from the studies of Gill (1980) and Philips and Gill (1987). Gill (1980) in his pioneering work explained the stationary wave solutions induced by an isolated heat source, which was symmetric w.r.t the equator. He showed that the heat source generated equatorially trapped long Kelvin waves towards its east and Rossby waves to its west. He demonstrated that at any given instant of time, the Kelvin waves had a greater zonal extent as compared to the Rossby waves, because of the faster speed associated with the Kelvin waves. Gill's model in
spite of its highly simplified formulation successfully explained many aspects of the heat induced Walker and Hadley-type circulations. However, the damping terms (Rayleigh friction and Newtonian cooling) in Gill's model were tunable parameters and the main shortcoming of this model was the use of strong dissipation terms (e-folding time \(~\sim 2\) days). In a similar study, Philips and Gill (1987), suggested that the scale of the east-west circulations was dependent on the strength of the dissipation terms. They argued that strong damping terms localized the response near to the heat source, while the use of weak damping terms resulted in stationary waves that extended far away from the heating region. However, at the present, the physical processes that produce large damping effects (e-folding time \(~\sim 2\) days) are not yet clearly understood. Therefore, it may not be desirable to employ large damping in models, in order to explain the scale and strength of the observed tropical east-west circulations. In our study we have considered the impact of the radiative cooling over the eastern Pacific, in addition to the convective forcing over Indonesia and western Pacific. The earlier idealized experiments of Gill (1980), Philips and Gill (1987) and many others, have not considered the influence of the radiative cooling over eastern Pacific in determining the scale and intensity of the time-mean Walker circulation. Our goal is to perform idealized forcing experiments, using a simple 2-level linear equatorial \(\beta\)-plane model and also a 5-level nonlinear global spectral model, to understand the nature of the time-mean response generated by a convective heat source in combination with a heat sink.

Another objective of our study is to determine the impact of nonlinear terms on the dynamics of the equatorially trapped Kelvin and Rossby waves. The theoretical studies of the Walker and Hadley circulations by Webster (1972), Gill (1982) and Lau and Lim (1982) are mostly based on linear models. There are, however, several reasons why nonlinear effects should be important at low latitudes. The first is the smallness of the Coriolis parameter, which tends to result in a greater overall influence
of the advective acceleration terms (i.e., a higher Rossby number). Additionally, a number of jet stream systems are present in the tropics, such as the subtropical, tropical easterly and Somali jets; these contain localized regions of very strong winds with significant nonlinear advections. There are other reasons for taking up the study of determining the influence of nonlinear terms on the dynamics of the stationary waves in the tropics. The earlier studies of the tropical time-mean response to heating, calculated by linear and nonlinear models showed some important differences. For example, in Gill-type of linear models, a strong Rayleigh friction had to be employed in order to obtain proper phase relation between the maximum upper level divergence and the geopotential distributions. In the absence of the strong dissipation, the upper level anticyclones in these models appeared on the western side of the heating region. However, Sardeshmukh and Held (1984), Sardeshmukh and Hoskins (1985) and Hendon (1986) have emphasized the role of nonlinearities on the tropical time-mean motions and demonstrated that the inclusion of nonlinearities can yield realistic upper tropospheric circulations even in the absence of strong dissipation terms. In the light of some of the above mentioned studies, we have re-examined the impact of nonlinearities on the stationary waves in the tropics. In order to highlight the role of nonlinear dynamics, we have adopted an approach where a comparison of the stationary solutions in the linear (LM) and nonlinear (NLM) versions of a 5-level global spectral model is made.

2.3 Linear steady state model formulation

The numerical 2-level linear model, on an equatorial $\beta$-plane, used for our studies is described below. We denote the zonal velocity, meridional velocity, geopotential and vertical $p$-velocity by $u, v, \phi$ and $\omega$ respectively. In the vertical, the equations
are discretized. The model levels are shown in Fig. 2.2. At levels 1 and 3, we apply the u-momentum equation, the v-momentum equation and continuity equation. The thermodynamic energy equation is applied at level 2. The following are the usual model equations:

\[ \frac{\partial u_1}{\partial t} - \beta y v_1 + \frac{\partial \phi_1}{\partial x} = 0 \]  
\[ \frac{\partial v_1}{\partial t} + \beta y u_1 + \frac{\partial \phi_1}{\partial y} = 0 \]  
\[ \frac{\partial u_3}{\partial t} - \beta y v_3 + \frac{\partial \phi_3}{\partial x} = 0 \]  
\[ \frac{\partial v_3}{\partial t} + \beta y u_3 + \frac{\partial \phi_3}{\partial y} = 0 \]

The continuity equation at level 1 gives,

\[ \omega_2 = - \Delta p \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) \]

\[ \frac{\partial \phi_3}{\partial t} - \frac{\partial \phi_1}{\partial t} - \sigma \Delta p^2 \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) = - \left( \frac{R \Delta p}{C_p \rho} \right) \dot{Q} \]

\[ \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) + \left( \frac{\partial u_3}{\partial x} + \frac{\partial v_3}{\partial y} \right) = 0 \]

In the model, the value of the static stability parameter \( \sigma \) is taken as \( 2.88 \times 10^{-6} \) \( \text{m}^2 \text{s}^{-2} \text{pa}^{-2} \) and \( \Delta p = 0.5 \times 10^5 \) pa. By choosing a velocity scale \( c = \sqrt{\frac{\sigma \Delta p^3}{2}} \) and a length scale \( L = \sqrt{\frac{c}{2\beta}} \) the following nondimensional quantities are defined:

\( x^* \left( \frac{x}{L} \right) \) ranges from \( -\frac{\pi a}{L} \) to \( \frac{\pi a}{L} \), 'a' being the earth's radius.

\( y^* \left( \frac{y}{L} \right) \) ranges from \( -\frac{\pi a}{4L} \) to \( \frac{\pi a}{4L} \), i.e., 45° S to 45° N.

\( t^* = \frac{tc}{L} \); \( u^* = \frac{u}{c} \); \( v^* = \frac{v}{c} \); and \( \phi^* = \frac{\phi}{c^2} \)

For convenience, we define \( \xi^* = \frac{L}{a} \left( x^* + \frac{\pi a}{L} \right) \) which ranges from 0 to 2\( \pi \). In order
Fig. 2.1. Dispersion relation for equatorial waves. The vertical axis is the frequency in units of $\sqrt{2\beta/c}$ and horizontal axis is the zonal wave number in units of $\sqrt{2\beta/c}$. (Adapted from A.F. Gill, Page 438, Atmosphere-Ocean Dynamics, Academic Press (1982)).

\[ \omega = 0 \]

0 mb

1

$\omega = 0$

250 mb

2

$\omega = 0$

500 mb

3

$\omega = 0$

750 mb

4

$\omega = 0$

1000 mb

Fig. 2.2. Vertical levels in the 2-level linear equatorial $\beta$-plane model.
to obtain steady state solution to the above forced problem, we set $\frac{\partial}{\partial t} = \varepsilon$. The nondimensional equations (after dropping the asterisk) are given by:

$$\varepsilon u_1 - \frac{yu_1}{2} + \frac{L}{a} \left( \frac{\partial \phi_1}{\partial \xi} \right) = 0 \quad (2.8)$$

$$\varepsilon v_1 + \frac{yu_1}{2} + \left( \frac{\partial \phi_1}{\partial y} \right) = 0 \quad (2.9)$$

$$\varepsilon u_3 - \frac{yu_3}{2} + \frac{L}{a} \left( \frac{\partial \phi_3}{\partial \xi} \right) = 0 \quad (2.10)$$

$$\varepsilon v_3 + \frac{yu_3}{2} + \left( \frac{\partial \phi_3}{\partial y} \right) = 0 \quad (2.11)$$

$$\varepsilon (\phi_3 - \phi_1) - 2 \left[ \frac{L}{a} \left( \frac{\partial u_1}{\partial \xi} + \frac{\partial u_1}{\partial y} \right) \right] = -H(\xi, y) \quad (2.12)$$

$$\frac{L}{a} \left( \frac{\partial u_1}{\partial \xi} + \frac{\partial u_3}{\partial \xi} + \left( \frac{\partial u_1}{\partial y} + \frac{\partial v_3}{\partial y} \right) \right) = 0 \quad (2.13)$$

The form of forcing with the heat source at $A \leq \xi \leq B$ and the heat sink at $C \leq \xi \leq D$ is shown in equation 2.14. The length of the heat source and the sink is 1 unit in the zonal direction. The value of $K$ is 0.07 so that the maximum heating (cooling) is $1^\circ$ per day.

$$H(\xi, y) = K (B - \xi) (\xi - A) \exp \left( -\frac{y^2}{4} \right) \text{ if } A \leq \xi \leq B$$

$$H(\xi, y) = -K (D - \xi) (\xi - C) \exp \left( -\frac{y^2}{4} \right) \text{ if } C \leq \xi \leq D \quad (2.14)$$

$$H(\xi, y) = 0 \text{ otherwise}$$

### 2.4 Linear steady state solution

We seek a solution that is periodic in the zonal direction and vanishes at the northern and southern boundaries. The variables are split into odd and even components
and expanded by a Fourier series in the zonal direction. In the meridional direction they are expanded using Parabolic Cylinder Functions $D_n(y)$ (Whittaker and Watson (1950)) as the basis function. For instance a variable $\psi(\xi, y)$ is expanded as:

$$
\psi(\xi, y) = \sum_{n=0}^{NM} \sum_{m=0}^{MM} [\psi(\epsilon)^{n\epsilon} \cos m\xi + \psi(\epsilon)^{n\epsilon} \sin m\xi] D_n(y) \quad (2.15)
$$

We have truncated the above series expansion at MM = 10 and NM = 6. The $D_0(y)$ component of the heat source and heat sink shown in 2.14 is symmetric w.r.t the equator. It now follows from Gill's (1980) calculations, that it would in fact be sufficient to truncate the meridional expansion at NM = 2. This is because the higher meridional wavenumbers associated with the Kelvin and Rossby waves forced by an idealized symmetric heat source are negligibly small.

$$
\varepsilon u_1(\epsilon)^m_n - [v_1(\epsilon)^m_{n-1} + (n + 1) v_1(\epsilon)^m_{n+1}] + \frac{mL}{a} \phi_1(\epsilon)^m_n = 0 \quad (2.16)
$$

$$
\varepsilon u_1(\epsilon)^m_n - [v_1(\epsilon)^m_{n-1} + (n + 1) v_1(\epsilon)^m_{n+1}] - \frac{mL}{a} \phi_1(\epsilon)^m_n = 0 \quad (2.17)
$$

$$
\varepsilon v_1(\epsilon)^m_n + [u_1(\epsilon)^m_{n-1} + (n + 1) u_1(\epsilon)^m_{n+1}]
+ \frac{1}{2} [(n + 1) \phi_1(\epsilon)^m_{n+1} - \phi_1(\epsilon)^m_{n-1}] = 0 \quad (2.18)
$$

$$
\varepsilon v_1(\epsilon)^m_n + [u_1(\epsilon)^m_{n-1} + (n + 1) u_1(\epsilon)^m_{n+1}]
+ \frac{1}{2} [(n + 1) \phi_1(\epsilon)^m_{n+1} - \phi_1(\epsilon)^m_{n-1}] = 0 \quad (2.19)
$$

$$
\varepsilon u_3(\epsilon)^m_n - [v_3(\epsilon)^m_{n-1} + (n + 1) v_3(\epsilon)^m_{n+1}] + \frac{mL}{a} \phi_3(\epsilon)^m_n = 0 \quad (2.20)
$$

$$
\varepsilon u_3(\epsilon)^m_n - [v_3(\epsilon)^m_{n-1} + (n + 1) v_3(\epsilon)^m_{n+1}] - \frac{mL}{a} \phi_3(\epsilon)^m_n = 0 \quad (2.21)
$$

$$
\varepsilon v_3(\epsilon)^m_n + [u_3(\epsilon)^m_{n-1} + (n + 1) u_3(\epsilon)^m_{n+1}]
+ \frac{1}{2} [(n + 1) \phi_3(\epsilon)^m_{n+1} - \phi_3(\epsilon)^m_{n-1}] = 0 \quad (2.22)
$$

$$
\varepsilon v_3(\epsilon)^m_n + [u_3(\epsilon)^m_{n-1} + (n + 1) u_3(\epsilon)^m_{n+1}]
+ \frac{1}{2} [(n + 1) \phi_3(\epsilon)^m_{n+1} - \phi_3(\epsilon)^m_{n-1}] = 0 \quad (2.23)
$$
The wavenumber components of the thermal forcing are given by

\[
\varepsilon [\phi_3 (\varepsilon)^m_n - \phi_1 (\varepsilon)^m_n] - \frac{2Lm}{a} u_1 (\varepsilon)^m_n.
\]

\[
-[(n + 1) v_1 (\varepsilon)^m_{n+1} - v_1 (\varepsilon)^m_{n-1}] = -H (\varepsilon)^m_n
\]  

(2.24)

\[
\varepsilon [\phi_3 (\varepsilon)^m_n - \phi_1 (\varepsilon)^m_n] + \frac{2Lm}{a} u_1 (\varepsilon)^m_n
\]

\[
-[(n + 1) v_1 (\varepsilon)^m_{n+1} - v_1 (\varepsilon)^m_{n-1}] = -H (\varepsilon)^m_n
\]  

(2.25)

\[
\frac{Lm}{a} [u_1 (\varepsilon)^m_n + u_3 (\varepsilon)^m_n] + \frac{1}{2} [(n + 1) v_1 (\varepsilon)^m_{n+1} - v_1 (\varepsilon)^m_{n-1}]
\]

\[
+ \frac{1}{2} [(n + 1) v_3 (\varepsilon)^m_{n+1} - v_3 (\varepsilon)^m_{n-1}] = 0
\]  

(2.26)

\[
\frac{Lm}{a} [u_1 (\varepsilon)^m_n + u_3 (\varepsilon)^m_n] + \frac{1}{2} [(n + 1) v_1 (\varepsilon)^m_{n+1} - v_1 (\varepsilon)^m_{n-1}]
\]

\[
+ \frac{1}{2} [(n + 1) v_3 (\varepsilon)^m_{n+1} - v_3 (\varepsilon)^m_{n-1}] = 0
\]  

(2.27)

The wavenumber components of the thermal forcing are given by

\[
H (\varepsilon)^0_n = \left[ \frac{1}{\sqrt{2 \pi n!}} \right] \frac{1}{2 \pi} \int_0^{2\pi} \int_{y_s}^{y_n} H (\xi, y) D_n (y) \, d\xi \, dy
\]

\[
H (\varepsilon)^m_n = \left[ \frac{1}{\sqrt{2 \pi n!}} \right] \frac{1}{\pi} \int_0^{2\pi} \int_{y_s}^{y_n} H (\xi, y) D_n (y) \cos m\xi \, d\xi \, dy
\]  

(2.28)

\[
H (\varepsilon)^m_n = \left[ \frac{1}{\sqrt{2 \pi n!}} \right] \frac{1}{\pi} \int_0^{2\pi} \int_{y_s}^{y_n} H (\xi, y) D_n (y) \sin m\xi \, d\xi \, dy
\]

where \(y_s\) and \(y_n\) are the southern and northern boundaries respectively. The system of linear algebraic equations (2.16)-(2.27) is solved by Singular Value Decomposition Method, Numerical Recipes (1988).

### 2.5 The nonlinear global spectral model

A brief description of the the 5-level nonlinear global spectral model used in our studies is given below. The model formulation is based on Bourke (1974). This is a dry model having a Rhomboidal truncation at 15 waves. The model equations comprise
of nonlinear prognostic equations for vorticity, divergence, temperature and surface pressure. We have used a weak Rayleigh friction and Newtonian cooling having a damping time scale of 10 days. The initial condition for all the experiments corresponds to an atmosphere at rest. The model equations are integrated until a steady state is attained. The forcing is kept fixed throughout the time of integration. The model equations are integrated using semi-implicit time integration scheme described in Bourke (1974) using a time-step of 3600 seconds.

2.6 Experiments and Results

We shall now describe the different forcing experiments that were carried out using the 2-level linear equatorial $\beta$-plane model and the 5-level global spectral model. In order to make our results more concrete, we have also included some analytical calculations. The main stress in these experiments is to examine the time-mean response of the tropical atmosphere, when subjected to different forcing conditions.

2.6.1 Idealized forcing experiments using heat source and sink

The annual mean diabatic heating (see Hoskins et al. (1989)) reveals that the tropical atmosphere over the equatorial Pacific is characterized by strong convective heating over Indonesia and western Pacific and a substantial radiative cooling in the eastern Pacific. As mentioned before, our intention is to understand the influence of this heat source and heat sink combination in controlling the zonal scale and strength of the time-mean Walker circulation in the equatorial Pacific. In the model formulation of
Gill (1980), only the heat source was used for forcing the model. He made use of large Rayleigh friction and Newtonian cooling (e-folding time \(\sim 2\) days) in order to explain the scale of the Walker circulation. In our numerical experiments, the model is integrated with the forcing kept fixed throughout the time of integration, till a steady state is attained. For the purpose of obtaining a steady state, we have employed weak damping terms (e-folding time \(\sim 10\) days) in our model. To begin with, we have forced the 5-level nonlinear global spectral model with a strong isolated heat source, which is symmetric w.r.t the equator, located near Indonesia (Fig.2.3a). The heating has a vertical variation of the form 

\[
Q(p) = \sin \left( \frac{\pi p}{p_0} \right), \quad \text{where} \quad p_0 \text{ corresponds to the 1000 mb pressure value.}
\]

This function has a maximum value at 500 mb and is zero at \(p = p_0\) and \(p = 0\). The maximum amplitude of the forcing is \(4^\circ\) K per day. The heating is switched on and the model is integrated for about 50 days so as to achieve the steady state response. The time-mean zonal winds and velocity potential fields at 300 mb are shown in Figs.2.3b and 2.3c respectively. The east-west circulations at the upper level are characterized by easterlies (shaded) to the west of the heat source and westerlies (solid lines) to the east of the forcing. The response to the west of the forcing corresponds to the stationary Rossby waves and the response on the eastern side corresponds to the stationary Kelvin waves. It can be clearly seen that the stationary Kelvin and Rossby waves are latitudinally confined between \(20^\circ\)S and \(20^\circ\)N. The maximum easterly speed is about \(-10\) ms\(^{-1}\) and the westerly speed is \(8\) ms\(^{-1}\). Gill (1980) has offered an elegant explanation for the zonal extent of the Kelvin and Rossby waves. He explained the greater zonal extent of the Kelvin wave response as compared to the Rossby wave response, by demonstrating that at any given instant of time the Kelvin waves could carry information almost three times faster than the Rossby waves. The velocity potential at 300 mb (Fig.2.3c) illustrates regions of outflow and inflow at the upper level. By looking at the velocity potential maps, one can also infer the approximate regions of ascending and descending motions.
Fig. 2.3a. (Top) Horizontal distribution of heat source over Indonesia. Interval 1 °K per day.

Fig. 2.3b. (Bottom) Latitude-longitude section of zonal wind at 300 mb (ms⁻¹). Shaded contours (interval 2 units) are easterlies and solid lines are westerlies (interval 2 units). Forcing used as shown in Fig. 2.3a.
Fig. 2.3c. (Top) Latitude-longitude section of velocity potential at 300 mb ($10^5 \text{ m}^2\text{s}^{-1}$). Shaded contours are negative (interval 18 units) and solid lines are positive (interval 5 units).

Fig. 2.4a. (Bottom) Horizontal distribution of heat source over Indonesia and heat sink in south eastern Pacific (units $^\circ$K per day). Positive contours (interval 1 unit) and negative contours (interval 0.5 unit).
Regions of \(-\chi\) correspond to outflow and regions of inflow can be identified with \(+\chi\). From Fig. 2.3c, one notices that there is an intense outflow (shaded) over Indonesia, due to strong upward motions over the heat source. The positive contours in Fig. 2.3c indicate that there is upper level convergence in the eastern Pacific and over South America. The upper level westerlies associated with the Walker circulation can be seen extending even to the east of South America. However the westerly speeds are quite small in the region of eastern Pacific (1-2 ms\(^{-1}\)).

Now let us examine the stationary response generated by including the radiative cooling over the south-eastern Pacific. The horizontal distribution of the heat source and heat sink is shown in Fig. 2.4a. The heat sink has a maximum cooling of 2° K per day, and is not symmetric w.r.t the equator. It should be noted that the strength of the cooling is only half the intensity of the heat source. This form of thermal distribution is quite realistic. The vertical structure of the heat sink is similar to that of the heat source. The steady state zonal wind distribution at 300 mb (Fig. 2.4b) reveals the stationary Kelvin and Rossby waves originating from the heat source. An additional feature is the appearance of strong westerlies in the central and eastern Pacific, indicating the strengthening of the Walker circulation (max 16 ms\(^{-1}\)). It can be clearly seen that the heat sink limits the upper level westerly flow, which does not extend beyond the eastern Pacific. The velocity potential distribution at 300 mb (Fig. 2.4c) shows that there is a strong convergence (+\chi) of westerlies over eastern Pacific. Further, one can clearly recognize that the zone of maximum convergence has significantly shifted westward over southeastern Pacific. In the eastern Pacific, the gradient of velocity potential along the zonal direction is positive. Further, the contours are close to one another, indicating that the magnitude of the gradient of the velocity potential in the longitudinal direction is quite large. This large positive gradient in the divergent component of the circulation supports the occurrence of strong westerlies in the equatorial Pacific. We shall later illustrate that the strong
Fig. 2.4b. (Top) Same as Fig. 2.3b, except for forcing as in Fig. 2.4a. Interval (2 units for negative and 4 units for positive contours).

Fig. 2.4c. (Bottom) Same as Fig. 2.3c, except for forcing as in Fig. 2.4a. Interval (20 units for negative and 10 units for positive contours).
upper level westerlies in the equatorial Pacific, arise because of the superposition of the Kelvin waves originating from the heat source and Rossby waves emanating from the heat sink. The conclusion from the above experiments is that the longwave cooling in the eastern Pacific is an important factor that determines the zonal extent (approximately the location of the subsiding branch) and also the intensity of the time-mean Walker circulation in the equatorial Pacific.

In the next set of experiments, we have studied the consequences of decreasing the strengths of the heat source and sink. Fig.2.5a shows a weak heat source, symmetric w.r.t the equator, with a maximum heating of 1° K per day. The steady-state wind vectors, zonal winds and perturbation geopotential at 300 mb are shown in Figs.2.5b, 2.5c and 2.5d respectively. The geopotential at 300 mb shows a high at the equator, which is flanked by anticyclones towards the north and south. In general the stationary wave response is much weaker, which is indicated by the weak easterlies (max speed -2.9 ms⁻¹) to the west of the heat source and weak westerlies (max speed 2.4 ms⁻¹) to its east. The important point to be recognized here is the weakening of the Walker circulation in the equatorial Pacific. Now a weak heat sink (max cooling of 1° K per day), which is separated by about 112.5° from the heat source, is also included in the forcing (Fig.2.6a). Here both the source and sink are symmetric w.r.t the equator. The corresponding steady-state wind vectors, zonal winds and perturbation geopotential at 300 mb are shown in Figs.2.6b, 2.6c and 2.6d respectively. It can be seen that the upper level westerly winds have increased (max value 5 ms⁻¹) and lows have appeared on either side of the equator at about 150°E. The upper level lows near 150°E which are forced by the symmetric heat sink lead to stronger convergence of the upper level westerlies over this longitude. A comparison of Figs.2.5b and 2.6b, suggests that the Rossby wave response is practically unaffected, while the intensity of the westerlies near the central Pacific is stronger. However the strength of the upper level westerlies is still weak as compared to the case of the strong heat
Fig. 2.5a. Horizontal distribution of heat source. Interval 0.25 °K per day.

Fig. 2.5b. Horizontal wind vectors at 300 mb. Forcing used as in Fig. 2.5a.
Fig. 2.5c. Zonal wind at 300 mb. Interval 0.59 ms\(^{-1}\). Forcing used as in Fig. 2.5a.

Fig. 2.5d. Perturbation geopotential at 300 mb. Interval 17 m\(^2\)s\(^{-2}\).
Fig. 2.6a. Horizontal distribution of heat source and heat sink separated by 112.5° longitudes. Interval 0.22 °K per day.

Fig. 2.6b. Same as Fig. 2.5b, except for forcing as in Fig. 2.6a.
Fig. 2.6c. Same as Fig. 2.5c, except for forcing as in Fig. 2.6a. Interval 0.89 ms$^{-1}$.

Fig. 2.6d. Same as Fig. 2.5d, except for forcing as in Fig. 2.6a. Interval 20 m$^2$s$^{-2}$. 
source and heat sink. The main result to be noted is that the radiative cooling in the eastern Pacific, no matter how small, produces an enhancement of the Walker circulation over the central and eastern Pacific. It also leads to the confinement of the time-mean Walker circulation between the heat source and the heat sink.

2.6.2 Analytical calculations

We can analytically work out the time-mean response generated by the combination of a heat source and heat sink (both symmetric w.r.t the equator). This will provide insight into the dynamics of the stationary wave pattern and circulation features, especially in the region between the heat source and heat sink. Analytical expressions can be easily derived by considering the shallow water equations of Gill (1980). The perturbation quantities are expanded in terms of Parabolic Cylinder functions as in Gill (1980). The calculation of the Kelvin and Rossby wave responses induced by a source-sink combination is given below. The thermal forcing is given by 2.29.

\[ Q(x, y) = F(x) P_0(y), \text{ where} \]

\[
\begin{align*}
F(x) &= 0 \text{ if } x \leq A \\
F(x) &= K(x - A)(B - x) \text{ if } A \leq x \leq B \\
F(x) &= 0 \text{ if } B < x < C \\
F(x) &= -\nu K(x - C)(D - x) \text{ if } C \leq x \leq D \\
F(x) &= 0 \text{ if } x \geq D
\end{align*}
\]

where the value of \( K \) is chosen such that the maximum heating rate is 1° per day and \( \nu (0 \leq \nu \leq 1) \) is a parameter used to vary the magnitude of the cooling rate. We define \( q = u + p \) as in Gill (1980), where \( u \) is the perturbation zonal velocity and \( p \) is
the perturbation pressure.

The $Q_0$ component of forcing involves the Kelvin and Rossby wave responses. From equations 3.9 and 4.6 of Gill (1980), the Kelvin wave response is given by $q_0$ and the Rossby wave response is given by $q_2$. The equations governing them are,

\[
\frac{dq_0}{dx} + \varepsilon q_0 = -F(x) \tag{2.30}
\]

\[
\frac{dq_2}{dx} - 3\varepsilon q_2 = F(x) \tag{2.31}
\]

The solution of the above equations is obtained by following the treatment for first-order linear inhomogenous equations given by Bender and Orzag (1978).

\[
q_0(x) = a \exp(\varepsilon x) - \exp(-\varepsilon x) \int F(x) \exp(\varepsilon x) \, dx \tag{2.32}
\]

\[
q_2(x) = \beta \exp(3\varepsilon x) + \exp(3\varepsilon x) \int F(x) \exp(-3\varepsilon x) \, dx \tag{2.33}
\]

where $\alpha$ and $\beta$ are constants of integration. By using cyclic boundary conditions and matching solutions at the boundaries, we can determine the solution in different regions of the domain. We know that the stationary wave response within the forcing region consists of the free and the forced components and outside the forcing region, the stationary wave response corresponds to the free component alone.

The boundary conditions used by Gill (1980) explicitly make use of the condition that Kelvin waves do not progress to the west of the heat source and Rossby waves do not move to the east of the heat source. However, we do not explicitly impose the above mentioned boundary conditions used by Gill (1980). Since the Kelvin and Rossby wave dynamics are inherently governed by the equations 2.30 and 2.31, their solutions based on cyclic (periodic) boundary condition should consistently bring out the nature of the wave responses. We shall first calculate the stationary Kelvin wave response as shown below:

\[
q_0(x) = [g_1(x)] = a_1 \exp(-\varepsilon x) \quad \text{if} \, x \leq A
\]
\( q_0(x) = [g_2(x)] = \alpha_2 \exp(-\varepsilon x) \)

\[-\frac{K}{\varepsilon^3} \left[ (B - x)(x - A) \varepsilon^2 + \varepsilon(2x - A - B) - 2 \right] \quad \text{if} \ A \leq x \leq B \]

\( q_0(x) = [g_3(x)] = \alpha_3 \exp(-\varepsilon x) \quad \text{if} \ B \leq x \leq C \)

\( q_0(x) = [g_4(x)] = \alpha_3 \exp(-\varepsilon x) \)

\[-\frac{K \nu}{\varepsilon^3} \left[ (D - x)(x - C) \varepsilon^2 + \varepsilon(2x - C - D) - 2 \right] \quad \text{if} \ C \leq x \leq D \]

\( q_0(x) = [g_5(x)] = \alpha_5 \exp(-\varepsilon x) \quad \text{if} \ x \geq D \)

By matching the solutions at the boundaries, the constants \( \alpha_2, \alpha_3, \alpha_4 \) and \( \alpha_5 \) are first expressed in terms of \( \alpha_1 \).

From the condition \( g_1(A) = g_2(A) \),

\[ \alpha_2 = \alpha_1 + \frac{K \exp(\varepsilon A)}{\varepsilon^3} \left[ \varepsilon(A - B) - 2 \right] \]

From the condition \( g_2(B) = g_3(B) \),

\[ \alpha_3 = \alpha_1 + \frac{K}{\varepsilon^3} \left[ \exp(\varepsilon A) \left( \varepsilon(A - B) - 2 \right) - \exp(\varepsilon B) \left( \varepsilon(A - B) - 2 \right) \right] \]

From the condition \( g_3(C) = g_4(C) \),

\[ \alpha_4 = \alpha_1 + \frac{K}{\varepsilon^3} \left[ \exp(\varepsilon A) \left( \varepsilon(A - B) - 2 \right) - \exp(\varepsilon B) \left( \varepsilon(A - B) - 2 \right) \right] + \frac{K \nu}{\varepsilon^3} \left[ -\exp(\varepsilon C) \left( \varepsilon(C - D) - 2 \right) \right] \]

From the condition \( g_4(D) = g_5(D) \),

\[ \alpha_5 = \alpha_1 + \frac{K}{\varepsilon^3} \left[ \exp(\varepsilon A) \left( \varepsilon(A - B) - 2 \right) - \exp(\varepsilon B) \left( \varepsilon(A - B) - 2 \right) \right] + \frac{K \nu}{\varepsilon^3} \left[ -\exp(\varepsilon C) \left( \varepsilon(C - D) - 2 \right) + \exp(\varepsilon D) \left( \varepsilon(D - C) - 2 \right) \right] \]

We can now determine \( \alpha_1 \) by using the cyclic boundary condition \( g_1(X_0) = g_5(X_N) \), from which

\[ \alpha_1 = \frac{K \exp(-\varepsilon X_N)}{\left[ \exp(-\varepsilon X_0) - \exp(-\varepsilon X_N) \right] \varepsilon^3 \left[ \exp(\varepsilon A) \left( \varepsilon(A - B) - 2 \right) \right] \]
The pressure and velocity components of the flow arising out of the Kelvin wave response are given by,

\[ u = p = \frac{1}{2} q_0(x) \exp \left( -\frac{y^2}{4} \right) \]  
\[ v = 0 \]  
\[ w = \frac{1}{2} [\varepsilon q_0(x) + F(x)] \exp \left( -\frac{y^2}{4} \right) \]

Similarly, the calculations for the Rossby wave response are shown below,

\[ q_2(x) = [h_1(x)] = \beta_1 \exp (3\varepsilon x) \quad \text{if } x \leq A \]
\[ q_2(x) = [h_2(x)] = \beta_2 \exp (3\varepsilon x) \]
\[ -\frac{K}{27\varepsilon^3} \left[ (B - x)(x - A) 9\varepsilon^2 + 3\varepsilon (A + B - 2x) - 2 \right] \quad \text{if } A \leq x \leq B \]
\[ q_2(x) = [h_3(x)] = \beta_3 \exp (3\varepsilon x) \quad \text{if } B \leq x \leq C \]
\[ q_2(x) = [h_4(x)] = \beta_4 \exp (3\varepsilon x) \]
\[ + \frac{K \nu}{27\varepsilon^3} \left[ (D - x)(x - C) 9\varepsilon^2 + 3\varepsilon (C + D - 2x) - 2 \right] \quad \text{if } C \leq x \leq D \]
\[ q_2(x) = [h_5(x)] = \beta_5 \exp (3\varepsilon x) \quad \text{if } x \geq D \]

By matching the solutions at the boundaries the constants \( \beta_2, \beta_3, \beta_4 \) and \( \beta_5 \) are first expressed in terms of \( \beta_1 \).

From the condition \( h_1(A) = h_2(A) \),

\[ \beta_2 = \beta_1 + \frac{K \exp (-3\varepsilon A)}{27\varepsilon^3} [3\varepsilon (B - A) - 2] \]
From the condition $h_2(B) = h_3(B)$,

$$
\beta_3 = \beta_1 + \frac{K}{27\varepsilon^3} \left[ \exp(-3\varepsilon A) [3\varepsilon (B - A) - 2] - \exp(-3\varepsilon B) [3\varepsilon (A - B) - 2] \right]
$$

From the condition $h_3(C) = h_4(C)$,

$$
\beta_4 = \beta_1 + \frac{K}{27\varepsilon^3} \left[ \exp(-3\varepsilon A) [\varepsilon (B - A) - 2] - \exp(-3\varepsilon B) [3\varepsilon (A - B) - 2] \right] - \frac{K\nu}{27\varepsilon^3} \left[ - \exp(-3\varepsilon C) [3\varepsilon (D - C) - 2] \right]
$$

From the condition $h_4(D) = h_5(D)$,

$$
\beta_5 = \beta_1 + \frac{K}{27\varepsilon^3} \left[ \exp(-3\varepsilon A) [3\varepsilon (B - A) - 2] - \exp(-3\varepsilon B) [3\varepsilon (A - B) - 2] \right] - \frac{K\nu}{27\varepsilon^3} \left[ \exp(-3\varepsilon C) [3\varepsilon (D - C) - 2] - \exp(-3\varepsilon D) [3\varepsilon (C - D) - 2] \right]
$$

We can now determine $\beta_1$ by using the cyclic boundary condition $h_1(X_0) = h_5(X_N)$, from which

$$
\beta_1 = \frac{K \exp(3\varepsilon X_N)}{\left[ \exp(3\varepsilon X_0) - \exp(3\varepsilon X_N) \right] 27\varepsilon^3} \left[ \exp(-3\varepsilon A) [3\varepsilon (B - A) - 2] - \exp(-3\varepsilon B) [3\varepsilon (A - B) - 2] - \nu \exp(-3\varepsilon C) [3\varepsilon (D - C) - 2] + \nu \exp(-3\varepsilon D) [3\varepsilon (C - D) - 2] \right]
$$

The flow fields due to the Rossby wave response are given below,

$$
u = \frac{1}{2} q_2(x) \left( y^2 - 3 \right) \exp \left( \frac{-y^2}{4} \right) \quad (2.37)
$$

$$
p = \frac{1}{2} q_2(x) \left( y^2 + 1 \right) \exp \left( \frac{-y^2}{4} \right) \quad (2.38)
$$

$$
v = [F(x) + 4\varepsilon q_2(x)] y \exp \left( \frac{-y^2}{4} \right) \quad (2.39)
$$
\[ w = \frac{1}{2} \left[ F(x) + \varepsilon q_2(x) \left( 1 + y^2 \right) \right] \exp \left( \frac{-y^2}{4} \right) \quad (2.40) \]

We notice that the region between the heat source and the heat sink contains both free Kelvin waves originating from the heat source as well as free Rossby waves originating from the heat sink. The time-mean geopotential at the upper level is characterized by highs near the region of the heat source and lows near the region of the heat sink. The Kelvin wave response of the heat source results in upper level westerlies to the east of the heat source. Similarly, the Rossby wave response of the heat sink produces westerly winds to its west. The lows to the west of the heat sink (Rossby wave response of the heat sink) lead to stronger convergence and strengthening of the westerlies in the region between the heat source and heat sink. This is essentially a superposition of free Kelvin waves (from the heat source) and free Rossby waves (from the heat sink) which leads to reinforcement of the westerly winds at the upper troposphere. Therefore, it will be more appropriate to view the time-mean Walker circulation in the equatorial Pacific as a stationary response resulting from a combination of the Kelvin waves (from the heat source) and Rossby waves (from the heat sink). Apparently the dynamics of this combined wave response will depend on the intensities of the heat source and the heat sink and also on the separation between them.

2.6.3 Sensitivity studies with the 2-level model

We have used the simple 2-level linear equatorial $\beta$-plane model, to study the changes in the time-mean Walker circulation, caused by zonal displacements in the relative positions of the Indonesian heat source and cooling in the eastern Pacific. Krueger and Winston (1974) reported that the Walker circulation in the equatorial Pacific was sensitive to the fluctuations in the spatial distribution of the radiative cooling
zone in the eastern Pacific. They found that during the years, when the heat sink
had a greater westward extent, the east-west circulation over the equatorial Pacific
exhibited considerable strengthening. We have the following three cases of forcings
(Figs.2.7a, 2.8a and 2.9a). In the first case, there is only a heat source (max heating
1° per day) symmetric w.r.t equator. In the second case, a symmetric heat sink
(max cooling 1° per day) is situated at a distance of 171° longitude from the heat
source. In the third case, the angular separation between the source and sink is
further increased to 228° longitude. Figs.2.7b, 2.8b, and 2.9b represent the zonal
wind at 250 mb for the three cases. Similarly, Figs.2.7c, 2.8c, and 2.9c represent
the perturbation geopotential at 250 mb for the three cases. In the first case, the
Rossby and Kelvin wave responses are seen in the zonal wind field, which is also
supported by the geopotential field. In the second case, we notice an increase in the
westerly speed in the region between the source and the sink and also the appearance
of a low (Rossby wave response of the sink) to the west of the heat sink. We have
already shown that this intensification of the Walker circulation is the result of the
superposition of Kelvin waves from the heat source and Rossby waves from the heat
sink. In the third case, when the separation between the source and the sink is further
increased, we can observe a decrease in the upper level westerly wind speed. This
weakening may be interpreted in the following way. When the distance between the
heat source and the heat sink is as large as 228°, the easterly regime (Rossby waves
from the heat source and Kelvin waves from the heat sink) starts dominating over
the westerly flow in the region between the source and the sink. This essentially leads
to a decrease in the upper level westerly flow and consequently a weakening of the
Walker circulation in the equatorial Pacific. These simple sensitivity experiments,
demonstrate that the changes in the relative separation between the heat source and
heat sink can considerably modify the longitudinal scale and strength of the east-west
circulations.
Fig. 2.7a. Horizontal distribution of heat source. Interval 0.11 °K per day.

Fig. 2.7b. Zonal wind at 250 mb, in the 2-level linear model. Interval 0.25 ms⁻¹. Forcing used as in Fig. 2.7a.
Fig. 2.7c. Perturbation geopotential at 250 mb. Interval 9.47 m$^2$s$^{-2}$. Forcing used as in Fig. 2.7a.

Fig. 2.8a. Horizontal distribution of heat source and heat sink separated by 171° longitudes. Interval 0.22 °K per day.
Fig. 2.8b. Same as Fig. 2.7b, except for forcing as in Fig. 2.8a. Interval 0.4 ms$^{-1}$.

Fig. 2.8c. Same as Fig. 2.7c, except for forcing as in Fig. 2.8a. Interval 13 m$^3$s$^{-2}$. 
Fig. 2.9a. Same as Fig. 2.8a, except for separation being 228° longitudes.

Fig. 2.9b. Same as Fig. 2.8b, except for forcing as in Fig. 2.9a. Interval 0.39 ms⁻¹.
Fig. 2.9c. Same as Fig. 2.8c, except for forcing as in Fig. 2.9a. Interval $13 \text{ m}^2 \text{s}^{-2}$.

Fig. 2.10a. Horizontal distribution of heat source and heat sink separated by $134^\circ$ longitudes. Interval $0.22^\circ\text{K per day.}$
The effects of varying the Rayleigh friction term have also been studied. We have considered two different cases of damping, with the forcing remaining the same. The forcing (Fig.2.10a) shows a symmetric source and sink, separated by 134° longitude. The maximum diabatic heating and cooling rates are 1°K per day. In the first case, we choose a strong damping which corresponds to an e-folding time of 2.5 days. In the weaker damping case, the decay time-scale corresponds to 5 days. The zonal winds at the lower level, for the two cases are shown in Figs.2.10b and 2.10c respectively. It can be seen that in the case of the larger damping, the low-level easterly speed in the region of the source and the sink is smaller (-2.55 ms^{-1}) and in the case of the weaker damping the speed is (-3.34 ms^{-1}). We can also notice a substantial reduction in the westerly winds in the case of the strong damping. However, it can be literally seen that the longitudinal scale of the easterlies is essentially the same in the two cases although the strength of the circulation is weak for case of strong dissipation. The implication is that the scale of the Walker circulation in the equatorial Pacific crucially depends on the separation between the heat source and sink.

2.6.4 Effects of nonlinearities on the stationary waves

The final question that we wish to address is the impact of nonlinear terms on the equatorially trapped stationary Kelvin and Rossby waves. The significance of nonlinear advection terms in determining the time-mean response in the tropics has been pointed by Sardeshmukh and Held (1984), Sardeshmukh and Hoskins (1985) and Hendon (1986). Our approach to study the impact due to nonlinearities, is to compare the steady-state response from the linear (LM) and nonlinear (NLM) versions of the 5-level global spectral model. The linear model (LM) is constructed basically by omitting all the nonlinear terms in the model equations. Comparisons of equilibrium solutions of LM and NLM have been performed for two cases of forcing. In the first
Fig. 2.10b. Zonal wind at lower level. Interval 0.57 ms$^{-1}$. Forcing used as in Fig. 2.10a and $\epsilon$ corresponds to decay time of 2.5 days.

Fig. 2.10c. Same as Fig. 2.10b, except for $\epsilon$ corresponding to decay time of 5 days. Interval 0.74 ms$^{-1}$.
case, the forcing is an idealized heat source symmetric w.r.t the equator and in the second case the forcing is antisymmetric w.r.t the equator.

Fig.2.11a shows a strong heat source symmetric w.r.t the equator and has a maximum heating rate of 4°K per day. The heating has a vertical variation of the form $Q(p) = \sin \left( \frac{\pi p}{p_0} \right)$, where $p_0$ corresponds to the 1000 mb pressure value. This function has a maximum value at 500 mb and is zero at $p = p_0$ and $p = 0$. We have chosen weak Rayleigh friction and Newtonian cooling terms (damping time-scale ~ 10 days) in the LM and NLM, in order to ensure that the changes in the equilibrium solution produced by nonlinear terms do not get masked by the dissipation effects. Both the models were forced with the heat source shown in Fig.2.11a and the models were integrated until the steady-state was attained. We have analyzed the flow patterns at the 300 mb pressure level in the two cases. The perturbation geopotential at 300 mb for the NLM and LM are shown in Figs.2.11b and 2.11d respectively. The zonal wind field in the two cases are represented in Figs.2.11c and 2.11e. A close examination of the equilibrium solutions in the LM and NLM reveals significant differences in the vicinity of the forcing. These distinct changes that are noticed close to the heating region suggest that the forced component of the stationary waves is strongly influenced by the nonlinearities. Anticyclonic circulations at the tropical upper troposphere can be seen in Figs.2.11b and 2.11d. Within and to the west of the forcing region, one notices that the equatorial high extends more eastward in the NLM as compared to the LM. The eastward shift is very prominently seen to the west of the forcing region and close to the equator. For instance the contour value (400 m$^2$s$^{-2}$) associated with the equatorial high to the west of the forcing extends upto about 70°E in the NLM. The same contour extends only upto 50°E in the LM. It is clear that the nonlinear advection terms have produced an eastward shift of the upper level anticyclonic features, on the western side of the forcing region, by nearly 20° longitudes. As one proceeds meridionally away from the equator, the eastward shift of
Fig. 2.11a. (Top) Horizontal distribution of symmetric heat source. Interval 1 °K per day.
Fig. 2.11b. (Bottom) Perturbation geopotential at 300 mb in N.L.M. Interval 100 m²s⁻². Forcing used as in Fig. 2.11a.
Fig. 2.11c. (Top) Zonal wind at 300 mb in NLM. Interval 3 ms⁻¹. Forcing used as in Fig. 2.11a.

Fig. 2.11d. (Bottom) Same as Fig. 2.11b, except for LM.
Fig. 2.11e. (Top) Same as Fig. 2.11c, except for LM and interval for westerlies is 2 ms$^{-1}$.
Fig. 2.11f. (Bottom) Difference (NLM-LM) for zonal velocity at 300 mb. Interval 1 ms$^{-1}$. 
the highs in the NLM becomes smaller. For example, the anticyclonic vortices having value of 600 m$^2$s$^{-2}$ which are centered at around 15°S and 15°N show almost the same eastward extents in both the NLM and LM. The zonal winds at 300 mb (Figs.2.11c and 2.11e.) reveal that, within the forcing region the pattern of easterlies (Rossby waves) and westerlies (Kelvin waves) have an eastward displacement of about 8° longitudes in the NLM. The maximum easterly speed in both the models is the same (-13 ms$^{-1}$) while the westerlies are stronger in the NLM (max speed is 12 ms$^{-1}$) as compared to the LM (max speed 8 ms$^{-1}$). The difference (nonlinear - linear) fields, for the zonal velocity, perturbation geopotential and divergence at 300 mb are given by Figs.2.11f, 2.11g and 2.11h respectively. The difference fields indicate that the structures of the stationary Kelvin and Rossby waves near the heat source are considerably affected by nonlinear terms. Fig.2.11g shows highs (solid lines) within the forcing region and lows (shaded) located to the east and west of the high. It was earlier seen in the NLM, that there is an eastward advection of the upper level equatorial high within and to the west of the forcing region. Therefore, the anticyclonic flow over the forcing region becomes stronger in the NLM. Consequently the upper level divergence, right over the forcing region, is stronger in the NLM. In Fig.2.11h, we notice that there is strong divergence (solid lines) right over the forcing region and convergence (shaded) to its west. The maximum divergence over the heating region is $2.0 \times 10^{-6}$ s$^{-1}$ and the maximum convergence is $-1.4 \times 10^{-6}$ s$^{-1}$. There are also small changes, caused by nonlinearities, in the structure of the upper level divergence within the region of forcing. Linear Gill-type models, cannot explain the eastward shift of upper level highs without including strong linear damping terms. However, our results suggest that even in the presence of weak dissipation, the influence of nonlinear terms can give rise to realistic phase relationship between the upper level divergence and geopotential distributions. From Fig.2.11f, the easterlies (shaded) and westerlies (solid lines) are well supported in the difference fields for perturbation geopotential and divergence.
Fig. 2.11g. (Top) Difference (NLM-LM) for perturbation geopotential at 300 mb. Interval 36 m$^2$s$^{-2}$.

Fig. 2.11h. (Bottom) Difference (NLM-LM) for divergence at 300 mb. Interval (0.3x10$^{-6}$ S for negative and 0.5x10$^{-6}$ S$^{-1}$ for positive contours).
The effect on the Kelvin waves is apparent from the equatorial westerlies and the effect on the Rossby waves is seen from the easterlies in Fig.2.11f. We notice that there are equatorial easterlies within and to the west of the forcing region. Likewise equatorial westerlies can be seen to the east of the forcing region. The enhancement of the anticyclonic features, over the forcing region, caused by nonlinear terms results in substantial zonal outflows appearing in Fig.2.11f. The relative vorticity at 300 mb in the NLM and LM are shown in Figs.2.11i and 2.11j respectively. The difference field (NLM - LM) for the relative vorticity at 300 mb is shown in Fig.2.11k. Figs.2.11i and 2.11j very clearly depict an eastward shift of the vorticity features in the NLM. We find that the anticyclonic vorticity (shaded in the northern hemisphere) at the upper levels extends up to about 85°E in the NLM, while it extends up to about 70°E in the LM, indicating an eastward shift of about 15° longitudes in the NLM. Similarly the upper level cyclonic vorticity (solid lines in the northern hemisphere) also shows some eastward shift in the NLM. This nonlinear effect shows up in the difference field (Fig.2.11k) in the form of substantial anticyclonic circulation over the forcing region. To the east of the forcing region, we find that there is an increased upper level cyclonic circulation caused by nonlinear terms.

The vertical structure of the equilibrium solutions in the LM and NLM shows interesting results. The vertical profiles of the zonal wind field were constructed by horizontally averaging the flow at each vertical level. The horizontal averages have been performed over two domains, namely the Kelvin and the Rossby wave domains. The Kelvin wave domain essentially extends eastward from the centre of the heat source and the Rossby wave domain extends westward from the centre of the heat source. In the Kelvin wave domain the averaging in the zonal direction was performed between 67.5°E and 118°E and for the Rossby wave domain the averaging in the zonal direction was performed between 17°E and 67.5°E. In the meridional direction, the averaging was performed between 7°S to 7°N, for both the Kelvin and Rossby wave
Fig. 2.11i. (Top) Relative vorticity at 300 mb in the NLM. Interval $3\times10^{-6} S^{-1}$.
Fig. 2.11j. (Bottom) Same as Fig. 2.11i. except for LM.
Fig. 2.11k. Difference (NLM-LM) for relative vorticity at 300 mb. Interval $3 \times 10^{-6} S^{-1}$. 

**NONLINEAR - LINEAR**

**REL VORTICITY AT 300 MB**
domains. Figs.2.12a and 2.12b show the vertical profiles of the zonal wind averaged in the Kelvin and Rosby wave domains. From Fig.2.12a, it is seen that the response in the nonlinear model (dotted line) shows considerable difference from the linear response (solid line) at the lower and middle tropospheric levels. The linear response shows a gradual change from a low level easterly flow to a westerly flow in the middle and upper troposphere. However, in the nonlinear model the wind reversal in the vertical direction is more sharper in the lower and middle tropospheric levels. It is also seen in Fig.2.12a that the nonlinear terms tend to weaken the easterlies and strengthen the westerlies in the lower and middle troposphere. The vertical profile of zonal wind in the Rossby wave domain (Fig.2.12b), shows significant differences in the linear and nonlinear response at 500 mb. It can be noticed that the linear model shows easterlies having speed of -6 ms$^{-1}$ at 500 mb, while the nonlinear model shows westerlies having speed of about 3 ms$^{-1}$. There is a complete change in the direction of flow in the LM and NLM at 500 mb. The zonal velocities at 500 mb for the NLM and LM are shown in Figs.2.13a and 2.13b respectively. One can notice that there are remarkable differences in the linear and nonlinear responses. Easterlies (max speed 8 ms$^{-1}$) can be seen to the west of the heating region in the LM. However, there is hardly any easterly flow near the equator and to the west of the heating region in the NLM. In fact easterlies can be seen only at around 25$^\circ$N and 25$^\circ$S in the NLM. Further, from the nonlinear response it is seen that westerlies extend very much to the west of the forcing region. Also the westerlies in the NLM are stronger (max value 7.5 ms$^{-1}$) as compared to the LM (max value 4 ms$^{-1}$). It appears that the nonlinear terms favour the generation of westerlies at 500 mb, which is the level of maximum heating. The role of nonlinear terms in favouring the generation of westerlies may be crucial for teleconnections from the tropics to midlatitudes (Hoskins and Karoly (1981) and Lau and Lim (1984)). Lim and Chang (1983) and Zhang and Webster (1989) have shown that transient Rossby waves are less trapped in equatorial
Fig. 2.12a. Vertical profile of zonal velocity (Kelvin domain). Dotted line is for NLM and solid line is for LM.

Fig. 2.12b. Same as Figure 2.12a, except for Rossby domain.
Fig. 2.13a. (Top) Zonal velocity at 500 mb in NLM. Interval for westerlies is 2.5 ms$^{-1}$ and easterlies is -0.5 ms$^{-1}$.

Fig. 2.13b. (Bottom) Same as Fig. 2.13a, except for LM. Interval for westerlies is 1 ms$^{-1}$ and easterlies is -2.5 ms$^{-1}$.
westerlies. Therefore, a westerly basic flow near the equatorial regions may serve to radiate Rossby waves from the tropics to the extratropics. One can also notice in Fig. 2.12b, that a stronger vertical shear occurs between 500 and 300 mb levels, in the NLM. The studies of Lim and Chang (1983), Lim and Chang (1986) and Kasahara and Silva Dias (1986) highlight the importance of vertical shear in the mean zonal wind in generating barotropic modes which are necessary for teleconnection mechanisms. Therefore, the stronger vertical shear in the NLM at around 400 mb in Fig. 2.12b, might be crucial for the propagation characteristics of transient Rossby waves.

We have also investigated the effect of nonlinearities on the Kelvin and Rossby waves, induced by symmetric heat sources of different amplitudes. Figs. 2.14a and 2.14b correspond to the difference field (nonlinear - linear) for zonal wind at 300 mb for the case of a weak forcing (max heating 1°K per day) and a strong forcing (max heating 5°K per day). It can be easily recognized that the nonlinearities have a much weaker effect when the forcing amplitude is smaller. We also find that the eastward nonlinear advection of the upper level anticyclonic features is smaller in the case of the weaker forcing. Our results agree with the findings reported by Hendon (1986).

We shall now describe the steady-state response in the LM and NLM for the antisymmetric forcing (Fig. 2.15a). The maximum heating (cooling) to the north (south) of the equator is 4°K per day. Gill (1980) has shown that an antisymmetric heating generates a stationary Rossby wave response, within and to the west of the forcing, which is characterized by a Hadley type of circulation. The winds and perturbation geopotential at 300 mb for the NLM and LM are shown in Figs. 2.15b and 2.15c respectively. We can notice upper level anticyclonic circulation in the northern hemisphere and cyclonic circulation in the southern hemisphere. The flows in the northern and southern hemispheres are mirror images of one another. The central position of the anticyclone (cyclone) is around 15°N (S) in both the models. Another
Fig.2.14a. (Top) Difference (NLM-LM) for zonal velocity at 300 mb. Weak forcing case. Interval 0.05 ms$^{-1}$.

Fig.2.14b. (Bottom) Same as Fig.2.14a, except for the strong forcing case. Interval 1.2 ms$^{-1}$.
Fig. 2.15a. (Top) Horizontal distribution of antisymmetric heat source. Solid lines are positive and shaded contours are negative.

Fig. 2.15b. (Bottom) Same as Fig. 2.11b, except for the antisymmetric forcing as in Fig. 2.15a. Interval 360 m² s⁻².
important feature is the prominent cross-equatorial flow. At the upper levels, we notice a strong easterly flow to the north and a westerly flow to the south of the equator. From Figs. 2.15b and 2.15c, one can notice that there is an eastward shift of the upper level circulation features, on the western side of the forcing, in the NLM. Due to this eastward shift of the upper level anticyclone (cyclone) in the northern (southern) hemisphere, there is an increase in the anticyclonic (cyclonic) flow, over the forcing region, in the NLM. Consequently, the difference (nonlinear - linear) field for the zonal wind (Fig. 2.15d) shows strong westerlies (easterlies) at around 10°N (S) respectively. The maximum change in the zonal wind speed, due to nonlinear effect, is as large as 10 ms⁻¹. This substantial change is also noticed in the difference field for the perturbation geopotential (Fig. 2.15e). In the northern hemisphere, we can see that there are highs (solid lines) at around 160°E and 20°N, which lie within the region of forcing. To the west of the high there are lows (shaded) that extend in the east-west direction. In the southern hemisphere, the difference field is more or less the mirror image of the northern hemispheric flow pattern. Strong meridional gradients in the perturbation geopotential can be observed in Fig. 2.15e. The difference field for zonal wind (Fig. 2.15d) is consistent with the meridional gradient of perturbation geopotential and very roughly satisfy geostrophic relation. In brief, experiments with NLM and LM for the antisymmetric forcing show that the stationary Rossby waves close to the forcing region are significantly affected by the nonlinear advection terms. An eastward advection of upper level anticyclone (cyclone) in the northern (southern) hemisphere is prominently observed within and to the west of the forcing region.
Fig. 2.15c. (Top) Same as Fig. 2.15b, except for L.M. Interval 300 m$^2$s$^{-2}$.

Fig. 2.15d. (Bottom) Difference (NLM-L.M) for zonal velocity at 300 mb. Interval 3 ms$^{-1}$.
Fig. 2.15c. Difference (NLM-LM) for perturbation geopotential at 300 mb. Interval (80 m$^2$s$^{-2}$ for negative and 60 m$^2$s$^{-2}$ for positive contours).
2.7 Conclusions

We have performed a variety of experiments both in the framework of linear as well as nonlinear models, with a motivation to understand the time-mean Walker circulation in the equatorial Pacific. Besides the strong convective heating over Indonesia, it is found that the radiative cooling in the eastern Pacific is an important factor in controlling the time-mean Walker circulation. It has been shown that the longitudinal scale and intensity of the time-mean east-west circulation is crucially dependent on the geometrical location and intensity of the Indonesian heat source and the heat sink in the eastern Pacific. In the region between the heat source and heat sink there is a superposition of Kelvin waves originating from the heat source and Rossby waves originating from the heat sink. This superposition leads to a strengthening of the upper level westerly flow over the equatorial Pacific. There are observational findings (e.g., Krueger and Winston (1974)) which suggest that the strength of the time-mean Walker circulation in the equatorial Pacific exhibits interannual fluctuations which primarily arise due to longitudinal shifts in the heating and cooling regions. In our idealized experiments, we have shown that a westward displacement of the heat sink in the eastern Pacific leads to a strengthening of the time-mean divergent circulation in the equatorial Pacific. Our results also reveal that when the heating over Indonesia or the radiative cooling in the Pacific is small, the intensity of the Walker circulation falls down.

By comparing the results from a linear global spectral model and a nonlinear global spectral model, we have studied the impact of the nonlinear advection terms on the equatorially trapped stationary Kelvin and Rossby waves. It has been shown that nonlinearities affect the Kelvin and Rossby waves mostly near the region of strong diabatic heating. An outstanding effect observed to the west of the forcing region is that, the nonlinear advection terms displace the upper level anticyclones eastward.
As a result, there is an enhancement of the upper level divergence over the region of forcing in the NLM. We also find that the upper level divergence over the region of forcing shows minor structural differences within the region of forcing. Linear Gill-type models cannot explain the eastward shift of upper level highs without including strong linear damping terms. However, our results suggest that even in the presence of weak dissipation, the influence of nonlinear terms can give rise to realistic phase relationship between the upper level divergence and geopotential distributions. We also find that the impact of nonlinearities is dependent on the strength of the forcing. The nonlinear effects decrease very much when the forcing amplitude is reduced. When an antisymmetric heat source was used, the nonlinearities affected the Rossby waves on the western side of the forcing. At the upper levels, an eastward advection of the anticyclone (cyclone) was observed in the northern (southern) hemisphere. The vertical structure of the time-mean zonal wind (for the case of the strong symmetric heat source) showed significant differences between the linear and nonlinear responses especially in the mid tropospheric levels. It was found that nonlinearities appear to favour the generation of westerlies particularly at 500 mb, where the forcing was maximum. To the west of the forcing region, an enhancement of the vertical shear in the zonal wind at around 400 mb was seen in the NLM. The generation of the westerlies and also the enhancement of the vertical shear in the basic flow, which are caused by nonlinear terms, may be important for the propagation of transient Rossby waves from the tropics to the extratropics and the phenomenon of midlatitude teleconnection.